DEPARTMENT OF MECHANICAL ENGINEERING

UNIT NOTES

20ME404- KINEMATICS OF MACHINERY

DEPARTMENT OF MECHANICAL ENGINEERING

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SUBJECT: KINEMATICS OF MACHINERY

Unit 1-BASICS OF MECHANISMS

Kinematics

Kinematics is the branch of classical mechanics which describes the motion of points, bodies (objects) and systems of bodies (groups of objects) without consideration of the causes of motion.

Machine

"a machine is an apparatus for applying mechanical power, consisting of a number of interrelated parts, each having a definite function".

A machine in its simplest form may be defined ***as a device which receives energy and transforms it into some useful work.*** Machines are used to transform some available form of energy into some other form which is necessary to produce a specific operation or which it desires to produce. They must transmit both force and motion. A machine consists of a number of parts or bodies.

Practical Examples :

- ✓ Dynamo used in the bi-cycle can be taken as a machine as it converts the mechanical energy into electrical energy which powers the head-lamp of the bicycle.
- ✓ A crowbar together with its fulcrum forms a machine, which enables the muscular energy of a man to be employed in raising a heavy weight.
- ✓ Machine tools in workshop such as lathe, shaper, planer, *etc.*, are machines as they convert the electrical energy supplied to them into useful work. *e.g.* turning a rod, cutting threads, turning tapers, *etc.*

Resistant Body

A body is said to be resistant if it is capable of transmitting the required force with negligible deformation. A link need not necessarily be a rigid body, but it must be a resistant body.

Practical Examples :

(i) Belts, ropes and chains.

(ii) Liquids (oils) are used as links in hydraulic presses, hydraulic brakes and hydraulic jacks.

Link or Element

A link or an element is defined as that part of a machine which has motion relative to some other part. A link need not to be a single unit, but it may consist of several parts which are manufactured as separate units.

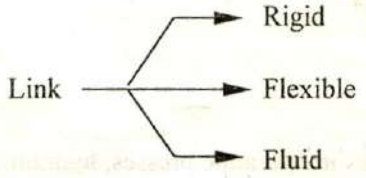
Characteristics of a Link :

(i) It should have relative motion, and

(ii) It must be a resistant body.

Types of Link

There are three types of links as classified below :



Rigid Link

A Rigid link is a link *which does not undergo any deformation* while transmitting motion. In the true sense, it is not possible to have a rigid link.

Practical Examples :

(a) Connecting rod and crank pin in a steam engine and (b) bed and spindle of a lathe do not have appreciable deflection and as such they can be termed as rigid links.

Flexible Link

A flexible link is partly deformed in a manner not to affect the transmission of motion.

Practical Examples :

Belts, ropes, chains, springs and bands are deformed considerably while transmitting motion but deformation has no effect on the transmission of motion.

Fluid Link

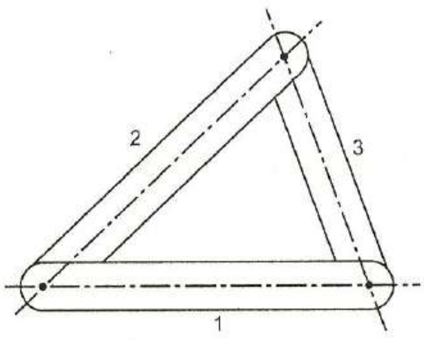
When motion is transmitted by means of a fluid, it is known as fluid link.

Practical Examples :

Fluids used in hydraulic press, hydraulic jack, hydraulic crane, *etc.*, acts as links.

Structure

Structure is an assemblage of number of resistant bodies having no relative motion between them. They are meant for carrying loads having straining action. The resistant bodies which constitute a structure are known as its members. Fig.1.2 shows sample structure having three members 1, 2 and 3.



Practical Examples :

Roof trusses, bridges, buildings, machine frames, *etc.*

Machine Vs Structure

S. No.	Machine	Structure
1.	Relative motion exists between its parts.	No relative motion exists between its members.
2.	It transforms available energy into useful work.	It does not convert the available energy into work.
3.	Links are meant to transmit motion and forces.	Members are meant for carrying loads having straining action.
4.	***Examples :*** Scooter, car, lathe, shaper, *etc.*	***Examples :*** Roof trusses, bridges, *etc.*

Kinematic Pair

A pair is a joint of two elements that permits relative motion. When any two links or elements are connected in such a way that their relative motion is completely or successfully constrained, they form a ***"Kinematic Pair"***.

Practical Examples :

In a reciprocating steam engine, the kinematic pairs existing are :

(a) Crank and connecting rod,

(b) Connecting rod and piston rod, and

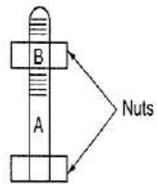
(c) Piston and engine cylinder.

Classifications

Based on Nature of contact or Type of contact

Lower Pair

If a pair in motion has a ***surface contact*** between the two elements, it is called a lower pair. It will be seen that sliding pairs, turning pairs and screw pairs form lower pairs.



Practical Examples :

Nut and bolt, Bolt and Socket joint, shaft rotating in bearing, piston reciprocating in a cylinder, *etc.*

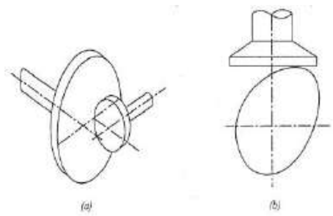
Higher Pair

If a pair in motion has a ***line or point contact*** between the two elements, it is called a higher pair. Pairs shown in Fig. are examples of higher pair

Practical Examples :

(i) Belt, rope and chain drives, and roller bearing, *etc.*, in which there is a line contact.

(ii) Gears, cam and follower, a pair of friction wheels and ball bearings, *etc.*, in which there is a point contact.

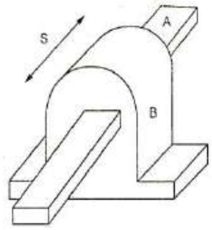


Classification Depends on Relative motion
Sliding Pair

(i) Sliding Pair :

When two elements have a sliding motion relative to each other, it is known as ***sliding pair.***

As shown in Fig. the bar A is constrained to have a sliding motion relative to bearing B, forms a sliding pair.

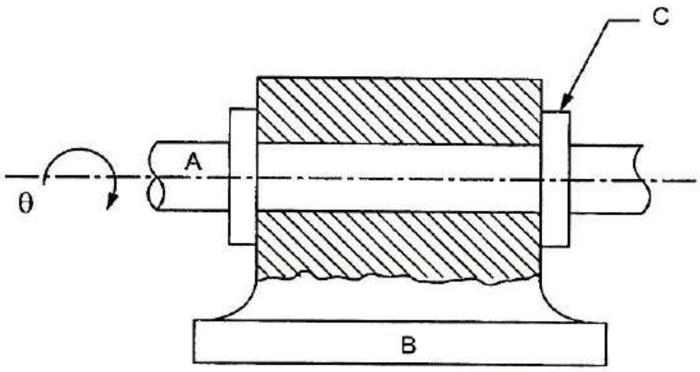


Practical Examples :

- ✓ Piston and cylinder,
- ✓ Cross-head and guides in a steam engine,
- ✓ Ram and its guides in a shaper, and
- ✓ Tail stock on the lathe bed.

(ii) Turning Pair

When two elements are connected such that one element revolves around the other, it forms a ***Turning pair.*** It is also called as ***Hinged Pair.***



As shown in Fig the shaft A with two collars rotates in a bearing B, forms a turning pair. Thus a turning pair has single degree of freedom.

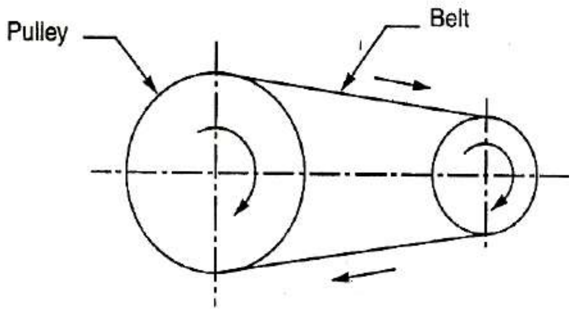
Practical Examples :

- ✓ Lathe spindle supported in the head stock,
- ✓ Crank shaft in a journal bearing in an engine,
- ✓ Cycle wheels turning over their axles, and
- ✓ Arbor supported between the arbor support and column of a milling machine.

(iii) Rolling Pair

When one element is free to roll over the other, it forms a rolling pair.

As shown in Fig. the belt driven pulley can be considered as a rolling pair.



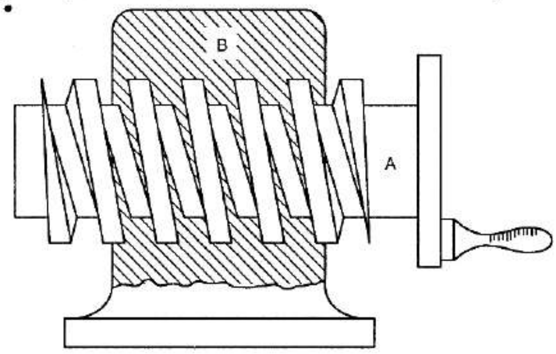
Practical Examples :

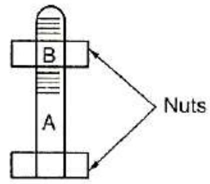
- ✓ Ball and roller bearings,
- ✓ A lawn mover rolling over a lawn, and
- ✓ A road roller rolling over the ground.

(iv) Screw Pair

In a screw pair, one link is constrained to have a combination of turning and sliding motion relative to the other element.

In Fig. , the elements A and B form a screw pair.





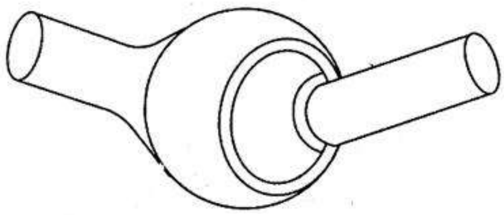
Practical Examples :

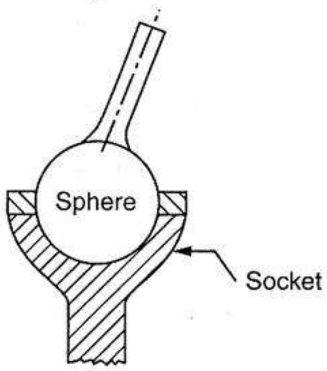
- ✓ Nut and bolt,
- ✓ Lead screw of a lathe with nut, and
- ✓ Threaded spindle and movable jaw in a bench vice.

(V) Screw Pair

In a spherical pair, one link is constrained to swivel in or about the other fixed point.

A Ball and socket joint as shown in Fig, represents the spherical pair.





Practical Examples :

- ✓ Attachment of a car mirror,
- ✓ Ball and socket joint, and
- ✓ Pen supporters joint in a pen stand.

Depend of Constraining Motion

(i) Closed Pair (Self-closed Pair) :

When two elements of a the pair are held together mechanically, they constitute a close pair. All the lower pairs are self closed pair.

Practical Examples : ✓ All lower pairs

(ii) Unclosed Pair (Open Pair or Force-closed Pair) :

When two links or elements are not held together mechanically, they constitute a unclosed pair.

Practical Examples :

- ✓ Cam and follower, and
- ✓ Flat belt running on a pulley.

In these cases, the contact between the two elements is maintained by the forces exerted by spring and gravity.

TYPES OF CONSTRAINED MOTIONS

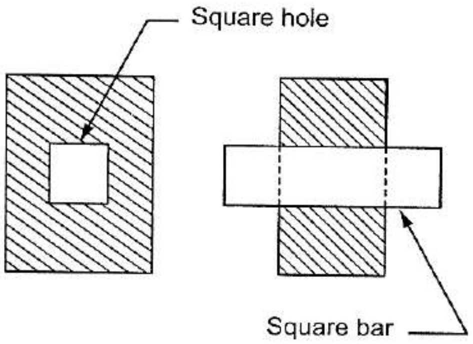
Constrained motions →
- Completely constrained motion
- Incompletely constrained motion
- Successfully constrained motion

Completely Constrained Motion

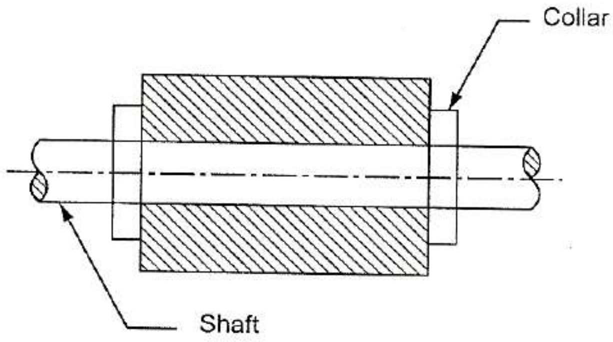
When the motion between a pair is limited to a definite direction, then the motion is said to be a completely constrained motion.

Examples :

- ✓ Square bar moving in a square hole
- ✓ A shaft with collars at its ends moving in a round hole.



(a) Square bar in a square hole

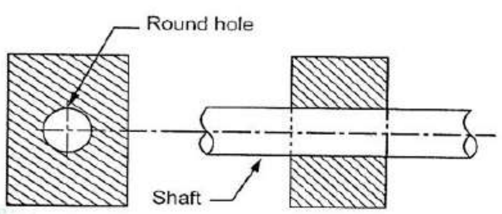


(b) Shaft with collars in a circular hole

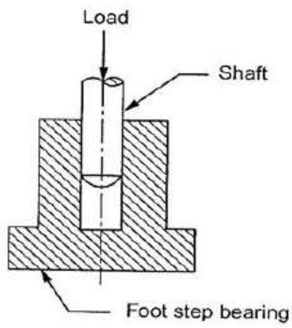
Incompletely Constrained Motion

When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion.

Examples :



(a) Shaft in a circular hole



(b) Shaft in a foot step bearing

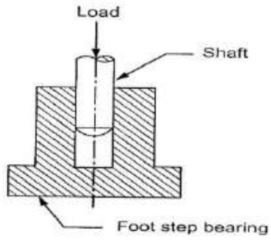
Successfully Constrained Motion

When the motion between the elements is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion.

Examples :

(i) The motion of piston inside the engine cylinder is not completely by itself but due to the rotation of crank. Similarly, the engine valves are not having their own motion but they are operated by rocker arms.

(ii) A shaft in a foot-step bearing, as shown in Fig. is also an example for successfully constrained motion.



Kinematic Chain

A Kinematic chain is defined as *a combination of kinematic pairs, joined in such a way that the relative motion between the links or elements is completely or successfully constrained.*

A chain may be locked, constrained and unconstrained.

Two equations for lower pairs are available to determine the assemblage of links and pairs forms the chain or not.

The two equations are :

$$\boxed{l = 2p - 4} \quad \ldots (1.1)$$

$$\boxed{j = \frac{3}{2}\, l - 2} \quad \ldots (1.2)$$

where l = number of links

p = number of pairs

j = number of joints

If above equations (1.1) and (1.2) are satisfied, then the assemblage of links form a kinematic chain.

Three possible cases are :

In equations (1.1) and (1.2),

(i) If **L.H.S. = R.H.S,** then the given chain is called ***constrained kinematic chain.***

(ii) If **L.H.S. > R.H.S,** then the given chain is called ***locked chain or structure.***

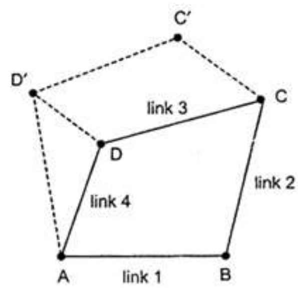
(iii) If **L.H.S. < R.H.S,** then the given chain is called ***unconstrained kinematic chain.***

Example:

From the figure, $l = 4$;

$p = 4$ (1,2) , (2,3) , (3,4) , (4,1)

and $j = 4$ (A, B, C, D)



From equation (1.1),

$$l = 2p - 4$$

$$4 = 2 \times 4 - 4 = 4$$

$$\boxed{\text{L.H.S.} = \text{R.H.S.}}$$

From equation (1.2),

$$j = \frac{3}{2} l - 2$$

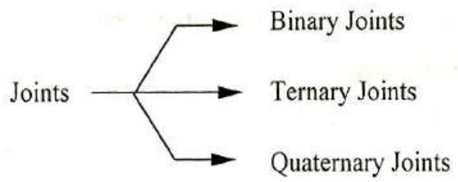
$$4 = \frac{3}{2} \times 4 - 2 = 4$$

$$\boxed{\text{L.H.S.} = \text{R.H.S.}}$$

Since the arrangement of four links satisfy the equations (1.1) and (1.2), and left hand side is equal to right hand side, therefore it is a ***kinematic chain.***

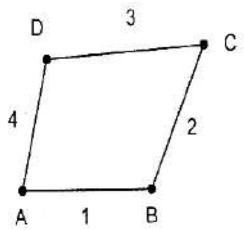
Joints

TYPES OF JOINTS



1. Binary joint

If two links are connected at the same joint, it is called a Binary Joint.



Here,

Number of Links = 4
Number of Binary Joints = 4

A.W. Klien's Criterion of Constraint :

A.W. Klien's Criterion of Constraint is used to determine the nature of chain, *i.e.*, whether the chain is a locked chain (or structure) or kinematic chain or unconstrained chain.

A.W. Klien's criterion of constraint is given by

$$\boxed{j + \frac{h}{2} = \frac{3}{2} l - 2} \qquad \ldots (1.3)$$

where, j = Number of binary joints

h = Number of higher pairs, and

l = Number of links

We can apply equation (1.3) to Fig.1.15,

$$j = 4; \quad n = 4 \text{ and } h = 0$$

$$j + \frac{h}{2} = \frac{3}{2} l - 2$$

$$4 + 0 = \frac{3}{2} \times 4 - 2 = 4$$

$$\text{R.H.S.} = \text{L.H.S.}$$

Thus the chain is a ***kinematic chain.***

DEGREE OF FREEDOM (OR) MOVABILITY

Degrees of freedom (DOF): It is the number of independent coordinates required to describe the position of a body in space.

It is also defined as the number of inputs **(number of independent coordinates)** required describing the configuration **or** position of all the links of the mechanism, with respect to the fixed link at any given instant.

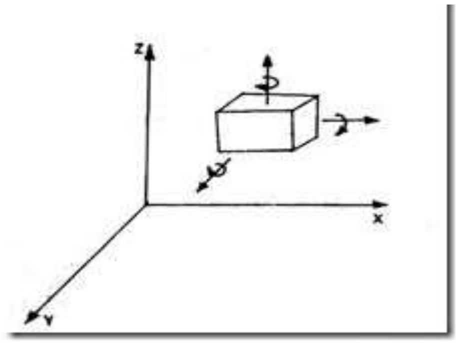


Figure 1.19.

A free body in space (fig 1.19) can have six degrees of freedom. i.e. linear positions along x, y and z axes and rotational/angular positions with respect to x, y and z axes. In a kinematic pair, depending on the constraints imposed on the motion, the links may lose some of the six degrees of freedom.

KUTZBACH CRITERION

kutzbach criterion for determining the number of degrees of freedom of movability (n) of a plane mechanism.

$$\mathbf{F=3(\mathit{n}-1)-2\mathit{l}-h}$$

Where n= no. of links
l=no. of binary joints
This equation is called kutzbach criterion for determining the number of degrees of freedom of movability (n) of a plane mechanism.

GRUBLER'S CRITERION FOR PLANE MECHANISM:

The Grubler's criterion applies to the mechanism to the only single degree of freedom joints where the overall movability of the mechanism is unity. Substituting F = 1 and h = 0 in kutzbach equation, we have,

$$1=3(n-1)-2l$$

$$1=3(n-1)-2l$$

or

$$3n-2l-4=0$$

$$3n-2l-4=0$$

This equation is known as the Grubler's criterion for mechanisms with constrained motion.

A little consideration will show that a plane mechanism with a movability of 1 and only single degree of freedom joints cannot have odd number of links. The simplest possible mechanism of this type are a four bar mechanism and a slider-crank mechanism in which l=4 and j=4.

EXAMPLES FOR DEGREE OF FREEDOM USING KUTZBACH CRITERION

$$\mathbf{F = 3(n-1)-2\mathit{l}-h.}$$

Where,

F = Degrees of freedom

n = Number of links = $n_2 + n_3 + \ldots\ldots + n_j$,

where, n_2 = number of binary links,

n_3 = number of ternary links…etc.

l = Number of lower pairs, which is obtained by counting the number of joints. If more than two links are joined together at any point, then, one additional lower pair is to be considered for every additional link.

h = Number of higher pairs

Examples of determination of degrees of freedom of planar mechanisms:

Example: 1:

Find the degree of freedom for the given arrangement shown in figure 1.20

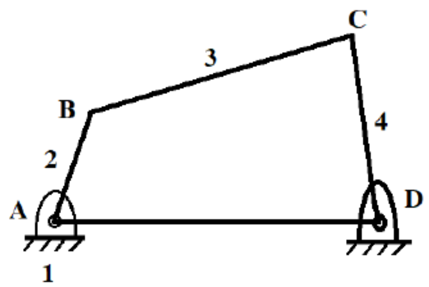


Figure 1.20.

F = 3(n-1)-2*l*-h

Here,

Number of links, n = 4,

Number of lower Pairs, l = 4

Number of higher Pair, h = 0.

F = 3(n-1)-2*l*-h

F = 3(4-1)-2(4) – 0 = 1

Degree of freedom, F= 1

i.e., one input to any one link will result in definite motion of all the links

Example :2

Find the degree of freedom for the given arrangement shown in figure 1.21

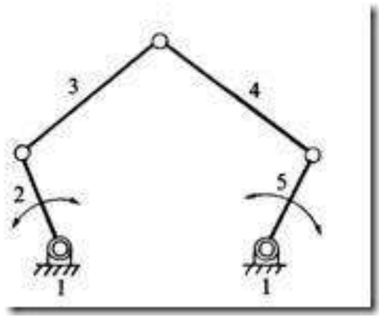


Figure 1.21.

F = 3(n-1)-2*l*-h

Here,

Number of links, n = 5,

Number of lower Pairs, *l* = 5

Number of higher Pair, h = 0.

F = 3(n-1)-2*l*-h

F = 3(5-1)-2(5) = 2 **Degree of freedom, F= 2**

i.e., two inputs to any two links are required to yield definite motions in all the links.

Example: 3

Find the degree of freedom for the given arrangement shown in figure 1.22

F = 3(n-1)-2*l*-h

Here,

Number of links, n = 6,

Number of lower Pairs, *l* = 7 (1,6) ,(6,5) , (5,3), (3,4), (1,4), (2,3), (2,1)

Number of higher Pair, h = 0.

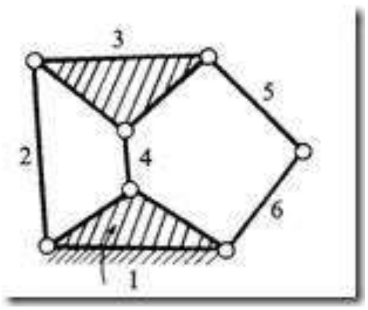


Figure 1.22.

F = 3(n-1)-2*l*-h

F = 3(6-1)-2(7) = 1 **Degree of freedom, F= 1**

i.e., one input to any one link will result in definite motion of all the links.

Example: 4

Find the degree of freedom for the given arrangement shown in figure 1.23

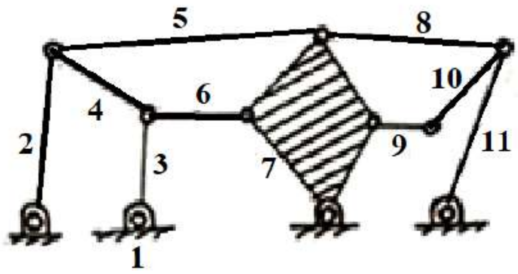


Figure 1.23.

F = 3(n-1)-2*l*-h

Here,

Number of links, n = 11,

Number of lower Pairs, *l* = 15

No.of binary joints = (1, 2) (1, 3) (1, 11), (9, 10) (7, 9) (1, 7) (6, 7) = 7

No.of ternary joints = (3, 4,6) ((2,4,5) (5,7,8) (8,10,11) = 4

So Number of binary joints = 1 x (No.of binary joints) + 2(No.of ternary joints)

= 1 (7) + 2(4)

l = 15

Number of higher Pair, h = 0.

F = 3(n-1)-2*l*-h

F = 3(11-1)-2(15) = 0 **Degree of freedom, F= 0** i.e., it's a structure, having no motion between them.

GRASHOF'S LAW

It **s**tates that for a four-bar linkage system, if the sum of length of shortest and longest of a planar quadrilateral linkage is less than or equal to the sum of the remaining two links , then the shortest link can rotate freely with respect to neighbouring link.

In a four bar chain there are four turning pairs and no sliding pairs. Let denote the smallest link of four bar linkage with S and the longest link by L and the other two links by P and Q as shown in figure 1.24.

The necessary condition to satisfy Grashof's Law is : **$S + L \leq P + Q$**

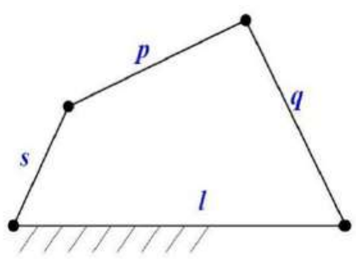


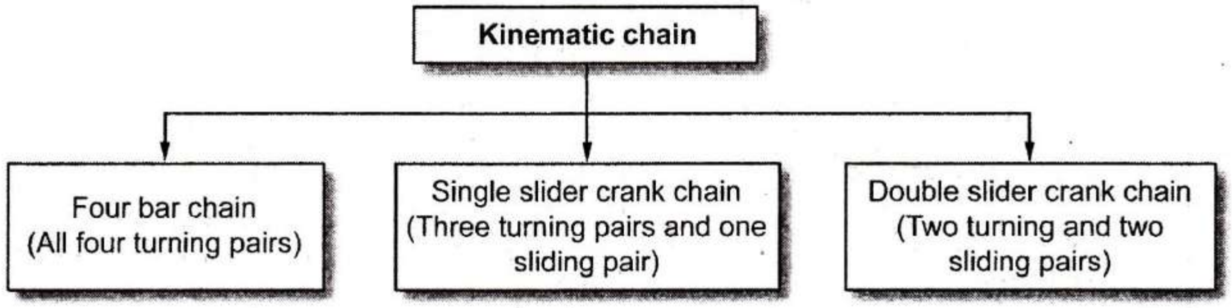
Figure 1.24. Four Bar Chain

Inversion Mechanism

Inversion of a kinematic linkage or mechanism is observing the motion of the members of the mechanism with fixing different links as reference frame. Each time when a different link is chose as the frame link the mechanism shows different characteristics of the motion.

The process of obtaining different mechanisms by fixing different links in a kinematic chain is known as inversion of the mechanism.

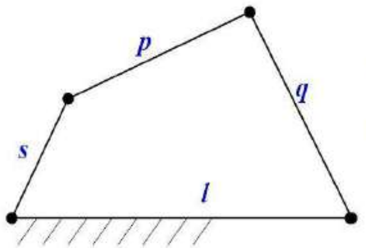
If a mechanism has 'n' number of links, then 'n' number of mechanisms can be obtained by fixing its different links one at a time.



Grashof's Law

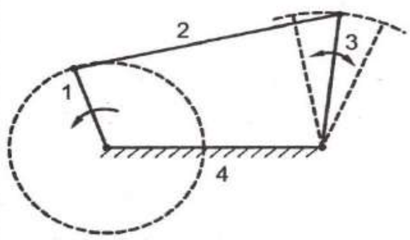
For a planar four-bar linkage, the sum of the shortest and longest link lengths cannot be greater than the sum of the remaining two link lengths if there is to be continuous relative rotational motion between two members.

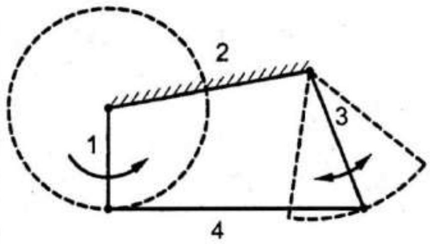
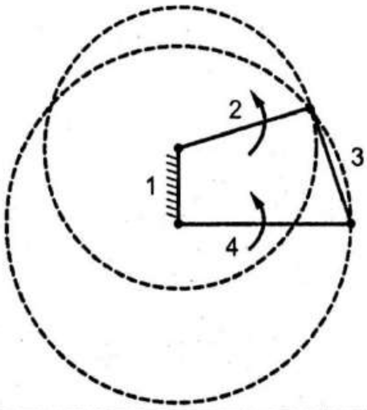
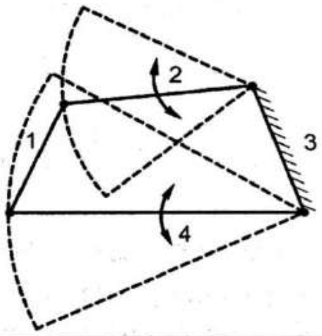
$$s + l \leq p + q$$



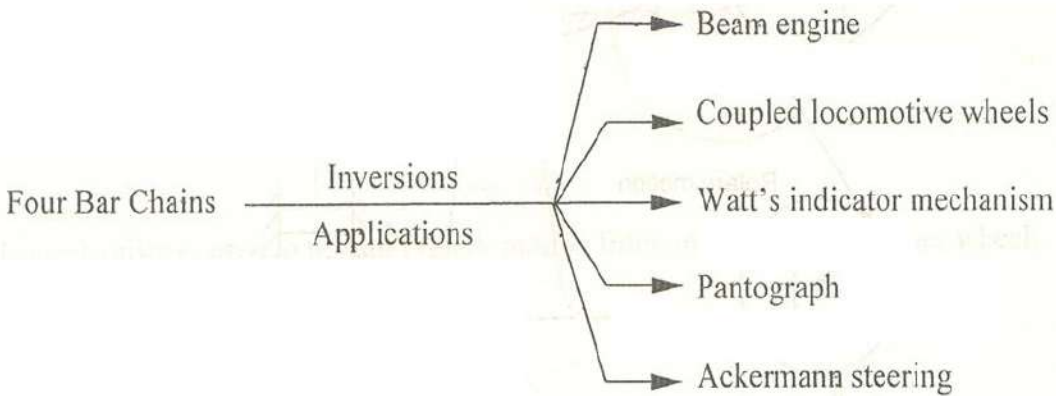
s – length of the shortest link
l – length of the longest link
p & q – the two remaining link length

Inversions of 4-Bar Chain

Sl.No.	Inversion	Mechanism	Practical Applications
1.	First inversion	***Crank-rocker mechanism*** Link 4 is fixed; link 1 rotates; links 2 and 3 oscillate.  	1. Beam engine 2. All rotary oscillating converters

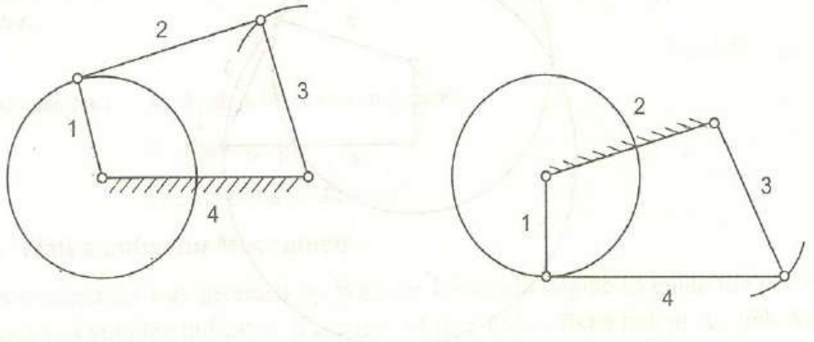
2.	Second inversion	***Crank-rocker mechanism*** Link 2 is fixed; link 1 rotates; links 3 and 4 oscillates 	1. Beam engine 2. All rotary oscillating converters
3.	Third inversion	***Double-crank mechanism*** Lin 1 is fixed; links 2 and 4 rotate; link 3 oscillates. 	1. Coupled wheels of a locomotive 2. All rotary-rotary converters
4.	Fourth inversion	***Double-rocker mechanism*** Link 3 is fixed; links 2 and 4 oscillate 	1. Watt's indicator mechanism 2. Pantograph 3. Ackermann steering

Inversions of 4 bar Chain



First Inversion (Crank and Lever Mechanism) :

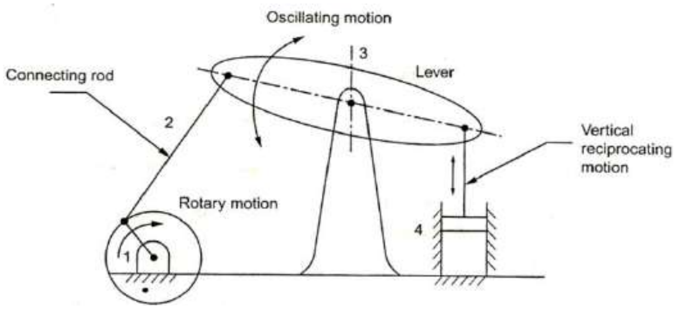
As shown in Fig. (a), link 1 is the crank, link 4 is fixed and link 3 oscillates whereas in Fig. (b), link 2 is fixed and link 3 oscillates. The mechanism is also known as ***crank-rocker mechanism or a crank-lever mechanism or a rotary-oscillating converter.***



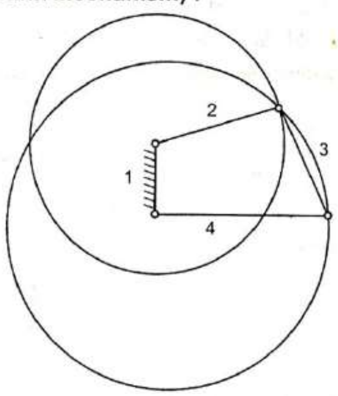
(a) (b)

Beam Engine :

This is an example of crank-lever mechanism, where one link oscillates, while the other rotates about the fixed link, as shown in Fig.



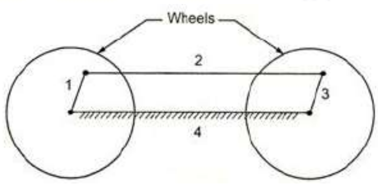
Second Inversion (Double Crank Mechanism) :



If the shortest link, *i.e.*, link 1 (crank) is fixed, the adjacent links 2 and 4 would make complete revolutions, as shown in Fig. The mechanism thus obtained is known as ***crank-crank*** or ***double crank mechanism*** or ***rotary-rotary converter.***

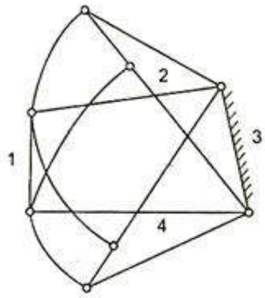
Application : Coupling rod of a locomotive.

This is an example of a double crank mechanism where both cranks rotate about the points in the fixed link. It consists of four links. The opposite links are equal in length, since links 1 and 3 work as two cranks, the mechanism is also known as ***rotary-rotary converter.***



Third Inversion :

If the link opposite to shortest link is fixed, *i.e.*, link 3 is fixed, then the shortest link (link 1) is made coupler and the other two links 2 and 4 would oscillate as shown in Fig. The mechanism thus obtained is known as ***rocker-rocker*** or ***double-rocker*** or ***double-lever mechanism*** or ***an oscillating-oscillating converter.***

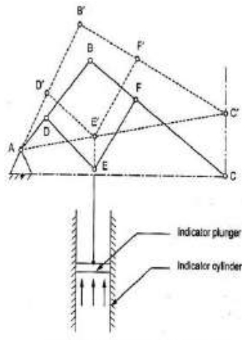


Applications :
1. Watt's indicator mechanism
2. Pantograph
3. Ackermann steering

Watt indicator Mechanism

This mechanism was invented by Watt for his steam engine to guide the piston rod. It is also known as simplex indicator. It consists of four links : fixed link at A, link AB, link BC and link DEF, connected to the piston of the indicator cylinder. Links BC and DEF work as levers and due to this, the mechanism is also known as double lever mechanism. The displacement of the lever DEF is directly proportional to the steam or gas pressure in the indicator cylinder.

In Fig. , full lines depict the initial position of the mechanism, whereas the dotted line shows the position of the mechanism when gas or steam pressure acts on the indicator plunger.



Watt's engine indicator (double lever mechanism)

Inversions of Single Slider Chain

Single slider crank chain — Inversions / Applications →
- Reciprocating engine
- Reciprocating compressor
- Whitworth quick return motion mechanism
- Rotary or Gnome engine
- Crank and slotted lever mechanism
- Oscillating cylinder engine
- Bull engine
- Hand pump

First Inversion

First inversion is obtained when the link 1 (*i.e.*, frame) is fixed and links 2 and 4 are made the crank and the slider respectively, as shown in Fig. (a).

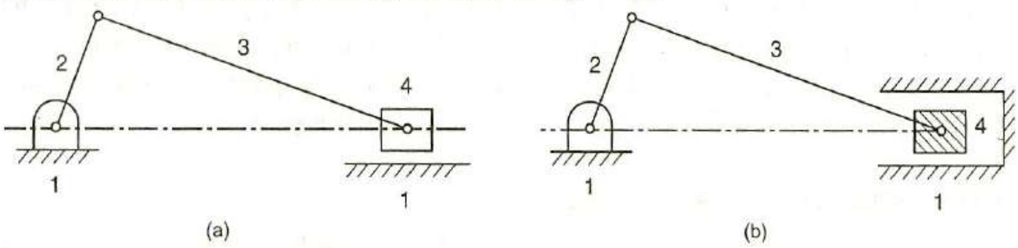


Fig.

Applications :
1. Reciprocating engine
2. Reciprocating compressor

Second Inversions

Second inversion is obtained by fixing the link 2 (*i.e.*, crank) of a slider crank chain. As shown in Fig. , when the link 2 is fixed, then the link 3 along with the slider at its end B becomes a crank. This makes link 1 to rotate about O along with the slider which also reciprocates on it.

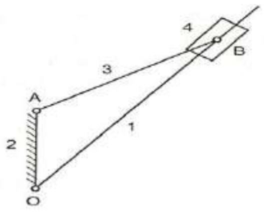


Fig.

Applications : 1. Whitworth quick-return mechanism

2. Rotary engine

Whitworth Quick-Return Mechanism :

It is a mechanism used in workshops *i.e.*, in shaping and slotting machines to cut metals. In this mechanism, link 2 (*i.e.*, crank) is fixed, link 3 rotates, link 4 reciprocates and link 1 oscillates as shown in Fig.

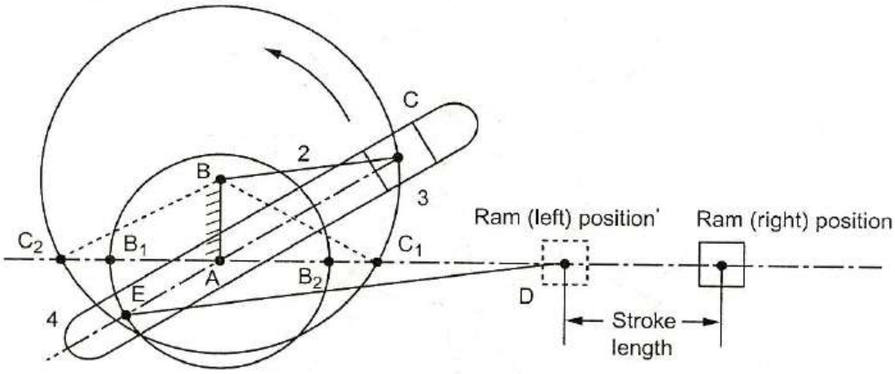


Fig. . Whitworth Quick Return Mechanism

Initially, let the slider 4 be at B_1 point C_2, then the tool will be in its extreme left position. When the crank further rotates and the slider 4 reaches to point C_1, the tool will be in its extreme right position. The distance between extreme left and right positions is the stroke length.

When the link AE rotates from the position AB_1 to AB_2, thenthe ram moves from left to right and corresponding movement of the crank link will be from BC_1 to BC_2. When the link AE further rotates, link AE moves from AB_2 to AB_1, then the ram moves from right to left and the crank BC correspondingly moves from BC_2 to BC_1.

Third Inversion

Third Inversion :

Third inversion is obtained by fixing the link 3 of the slider crank mechanism, as shown in Fig. . In this, link 2 again acts as a crank and link 4 oscillates.

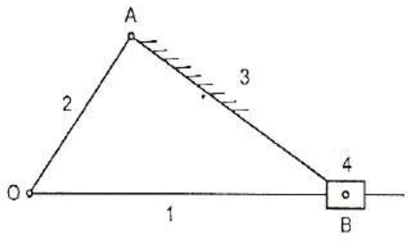


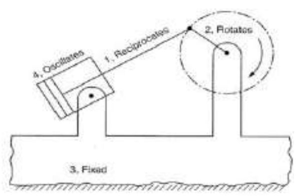
Fig.

Applications : 1. Oscillating cylinder engine, and

2. Crank and slotted lever mechanism.

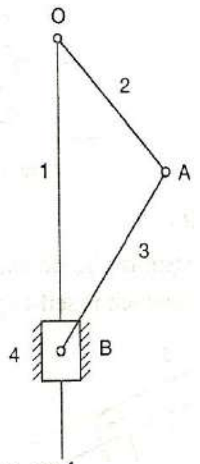
Oscillating Cylinder Engine :

The oscillating cylinder engine mechanism is used to convert reciprocating motion into rotary motion, as shown in Fig.



In this mechanism, link 3 is fixed. When the link 2 (crank) rotates, the piston attached to link 1 (piston rod) reciprocates and the link 4 (cylinder) oscillates about a pin pivoted to the fixed link at A.

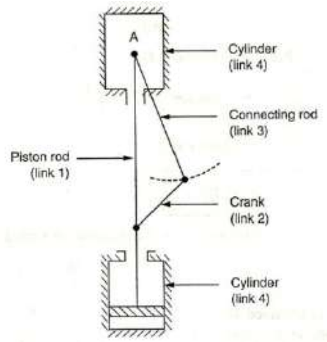
Fourth Inversion



Fourth inversion is obtained by fixing the link 4 of the slider crank mechanism, as shown in Fig. . In this inversion, link 3 can oscillate about the fixed pivot B on link 4. This makes end A of link 2 to oscillate about B and end O to reciprocate along the axis of the fixed link 4.

Applications :
1. Pendulum pump or Bull engine, and
2. Hand pump.

Pendulum Pump



This mechanism is obtained by fixing the link 4 (*i.e.*, cylinder), as shown in Fig. In this case, when link 2 (crank) rotates, link 3 (connecting rod) oscillates about a pin pivoted to the fixed link 4 at A and link 1 (piston) reciprocates. This mechanism is used to supply feed water to boilers.

INVERSIONS OF DOUBLE SLIDER CRANK CHAIN

Double slider crank chain —Inversions / Applications→
- Elliptical Trammel
- Scotch yoke mechanism
- Oldham coupling

First Inversion :

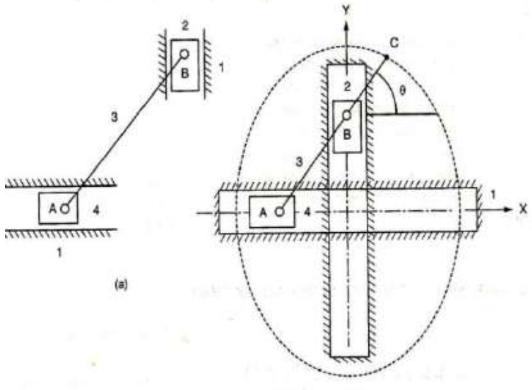
First inversion is obtained by fixing the link 1. In this, the two adjacent pairs 2 – 3 and 3 – 4 are turning pairs and the other two pairs 1 – 2 and 1 – 4 are sliding pairs. Refer Fig. (a).

Applications : Elliptical trammel

Elliptical Trammel :

Elliptical trammel is an instrument used for drawing ellipses. This inversion is obtained by fixing the link 1 (*i.e.*, slotted plate), as shown in Fig. (b). The link 1 or the fixed plate has two straight grooves in it, at right angles to each other.

When the links 2 and 4 (sliders) slide along their respective grooves, the end C of the extension BC of the link AB, traces an ellipse such that AC and BC are the semi-major and semi-minor axis of the ellipse respectively.



(a)

(b)

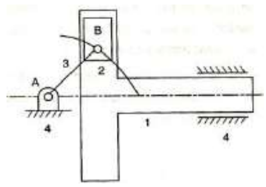
Second Inversion :

Second inversion is obtained by fixing any one of the slider blocks of the first inversion. When link 4 is fixed end B of crank 3 rotates about A and link 1 reciprocates in the horizontal direction.

Application : Scotch yoke mechanism

Scotch Yoke Mechanism :

This inversion is used for converting rotary motion into reciprocating motion. It is obtained by fixing any one of the sliders, as shown in Fig. As crank 3 rotates, the horizontal portion of link 1 slides or reciprocates in the fixed link 4.



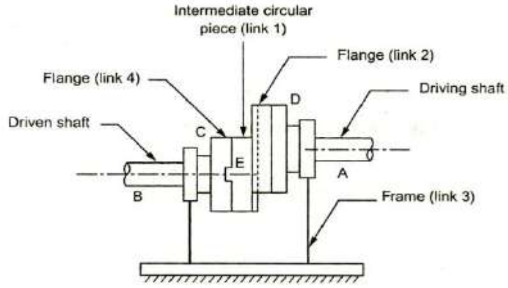
Third Inversion

Third inversion is obtained by fixing the link 3 of the first inversion. In this inversion link 1 is free to move.

Application : Oldham's coupling

Oldham's Coupling :

This mechanism is used for transmitting motion between two shafts which are parallel but not coaxial. This inversion is obtained by fixing the link 3 as shown in Fig. It consists of a driving shaft, fitted with a flange (link 2) having a diametrical slot on its face; a driven shaft fitted with flange C (link 4) also has diametrical slot on its face. This whole makes link 4. The slots on the two flanges are at right to each other. An intermediate piece circular shape and having tongues X and Y at right angles on opposite sides, is fitted in between the flanges of the two shafts in such a way that the tongues of the intermediate piece get fitted closely in the slots of flanges. The intermediate circular piece E forms link 1 which slides between the flanges C and D. When driving shaft rotates through certain angle the driven shaft also rotates through the same angle. Motion is transmitted through intermediate link 1. If the distance between the axis of the shafts is x, it will be the diameter of a circle traced by the center of intermediate piece.



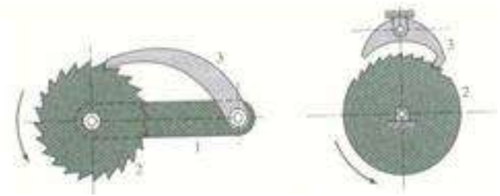
MECHANISMS

Toggle Mechanism
The toggle mechanisms can be used in the situation when one needs to output large force subject to a short stroke, for example, the stone crushers and mechanical presses, etc. The shown mechanism has a toggle position when the two lower links arrange to be aligned. At this position, the slider can produce an extremely large power to press workpiece.

Ratchet and Pawl Mechanism

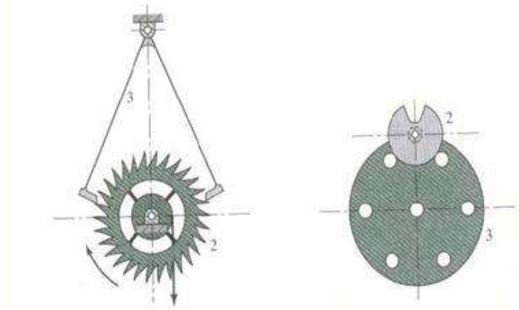
Ratchets

A ratchet is used to ensure that the motion of the output device is only allow in one direction even though the input motion may be in either direction or oscillatory.



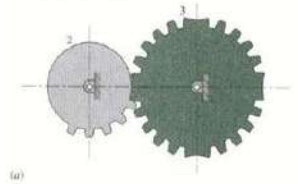
Escapement

Escapements are used for to control continuous motion to produce a highly controlled step motion at a fixed rate. Escapements are used for mechanically driven clocks. When used with clocks the escapement controls the spring driven clock mechanism such that it moves in regulated steps controlled by a pendulum or an oscillating arm.

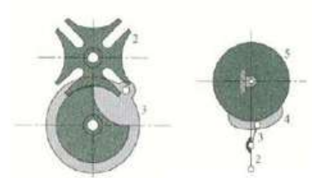


Indexing Mechanisms

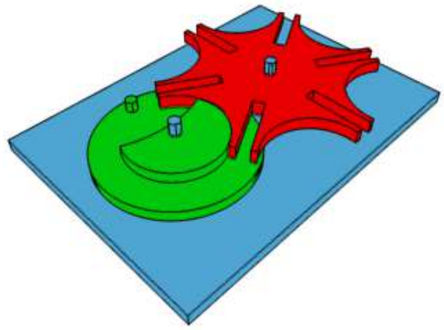
Indexing mechanisms generally converts a rotating, rocking or oscillatory motion to a series of step movements of the output link or shaft. Indexing mechanisms are useful for counters and machine tool feeds.



Geneva-wheel indexer

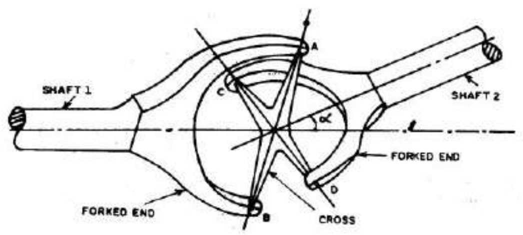


Geneva Wheel Mechanism



Geneva mechanism, also called **Geneva Stop**, one of the most commonly used devices for producing intermittent rotary motion, characterized by alternate periods of motion and rest with no reversal in direction. It is also used for indexing (*i.e.,* rotating a shaft through a prescribed angle).
In the Figure the driver A carries a pin or roller R that fits in the four radial slots in the follower B. Between the slots there are four concave surfaces that fit the surface S on the driver and serve to keep the follower from rotating when they are fully engaged. In the position shown, the pin is entering one of the slots, and, on further rotation of the driver, it will move into the slot and rotate the follower through 90°. After the pin leaves the slot, the driver will rotate through 270° while the follower dwells—*i.e.,* stands still. The lowest practical number of slots in a Geneva mechanism is 3; more than 18 are seldom used. If one of the slot positions is uncut, the number of turns that the driver can make is limited. It is said that the Geneva mechanism was invented by a Swiss watchmaker to prevent the over winding of watch springs. For this reason it is sometimes called a Geneva stop.

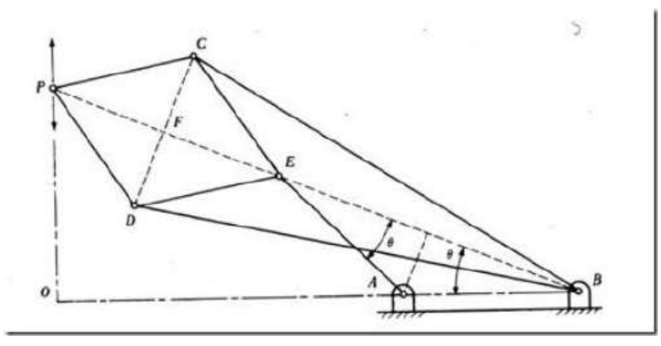
Universal Joint
A universal joint is a mechanical device that allows one or more rotating shafts to be linked together, allowing the transmission of torque and/or rotary motion. It also allows for transmission of power between two points that are not in line with each other.
Universal joints come in a wide variety of shapes, sizes and configurations to accommodate the infinite amount of applications they can go into. You should always consult a professional before selecting a universal joint.



STRAIGHT LINE MOTION MECHANISMS

Straight line motion mechanisms are mechanisms, having a point that moves along a straight line, or nearly along a straight line, without being guided by a plane surface.

Peaucellier exact straight line motion mechanism:



Peaucellier exact straight line motion mechanism

Here, AE is the input link and point E moves along a circular path of radius AE = AB. Also, EC = ED = PC = PD and BC = BD. Point P of the mechanism moves along exact straight line, perpendicular to BA extended.

To prove B, E and P lie on same straight line:

Triangles BCD, ECD and PCD are all isosceles triangles having common base CD and apex points being B, E and P. Therefore points B, E and P always lie on the perpendicular bisector of CD. Hence these three points always lie on the same straight line.

To prove product of BE and BP is constant.

In triangles BFC and PFC,

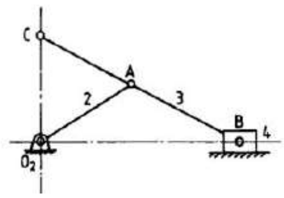
$$\therefore BC^2 - PC^2 = FB^2 - PF^2 = (FB + PF)(FB - PF) = BP \times BE$$

$BC^2 = FB^2 + FC^2$ and $PC^2 = PF^2 + FC^2$

But since BC and PC are constants, product of BP and BE is constant, which is the condition for exact straight line motion. Thus point P always moves along a straight line perpendicular to BA as shown in the fig

Scott Russel Straight Line Mechanism

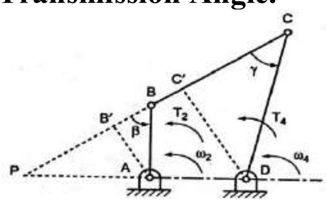
The complexity of the mechanisms to generate exact straight lines can be reduced by introduction of one or more slider crank linkages. It is possible to generate an exact straight line using the slider crank mechanism but the range of motion is limited. Based on the geometry of the linkage the output motion is a simple sine function of the drive link or a simple harmonic motion. It is evident from the figure that this mechanism is made up of isosceles triangles, AB, AC and AO2 are of equal lengths.



Mechanical Advantage

- Due to more usage of 4 bar mechanism, its necessary to study some of the advantages of mechanisms.
- Which tell whether the mechanism is good one or not.
- It's a quality measure of all mechanism

Transmission Angle.



β be the acute angle made by the coupler, or its extension and the driving link
γ be the acute angle made by the coupler and the driven link.

- The angle between Coupler and the follower

Effect of transmission Angle

As the value of transmission angle (γ) becomes small, the mechanical advantage decreases and even a small amount of friction will cause the mechanism to lock or jam.

A common rule of thumb is that a four bar linkage should not be used in the region where the transmission angle is less than 45°.

Unit 2- KINEMATICS OF LINKAGE MECHANISMS

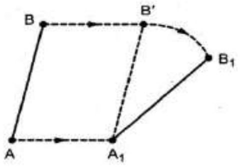
Instantaneous Centre of Rotation

The combined motion of rotation and translation of a link in the mechanism may be assumed to be a motion of pure rotation about some centre I, known as the ***instantaneous centre of rotation*** *or* ***virtual centre of rotation.***

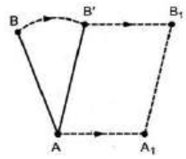
Example

Consider a rigid link AB which moves from its initial position AB to A_1B_1 as shown in Fig . AB has reached to A_1B' position through translatory and from A_1B' to A_1B_1 through rotary motion. In Fig. (b), the link AB has first the motion of rotation from AB to AB′ about A and then the motion of translation from AB′ to A_1B_1. Thus A_1B_1 which is the final position of AB, is the combined effect to translatory and rotary motions.

This combined motion of rotation and translation of the link AB may be assumed to be a motion of pure rotation about some centre I, known as the ***instantaneous centre of rotation.***



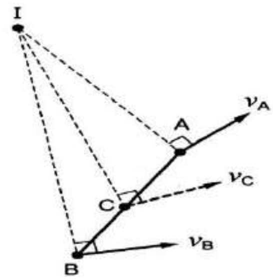
(a)



(b)

Velocity of a point on a link by Instantaneous Centre Method

Consider a link AB



Let ω be the angular velocity of link AB in the plane of rotation. Velocity of point A can be written as

$$v_A = IA \times \omega \quad \text{or} \quad \omega = \frac{v_A}{IA} \qquad \ldots (i)$$

and velocity of point B,

$$v_B = IB \times \omega \qquad \ldots (ii)$$

or $$v_B = IB \times \frac{v_A}{IA}$$

To find v_C: Let C be any point on the link AB. Then join C and I as shown in Fig

$$\text{Magnitude of } v_C = IC \times \omega \qquad \ldots (iv)$$

$$\text{Direction of } v_C = \text{Perpendicular to IC}$$

From equations (*i*), (*ii*) and (*iv*), we can write

$$\boxed{\frac{v_A}{IA} = \frac{v_B}{IB} = \frac{v_C}{IC} = \omega}$$

From equation , it can be stated that *the magnitudes of the velocities of the points on a link is directly proportional to their distance from the instantaneous centre. The direction of their velocities is perpendicular to the line joining them with the instantaneous centre.*

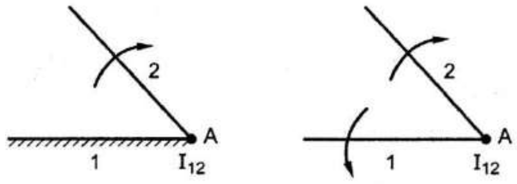
Number of Instantaneous Centers in a Mechanism

Mathematically, the number of instantaneous centres (N) in a mechanism is given by

$$\boxed{N = \frac{n(n-1)}{2}}$$

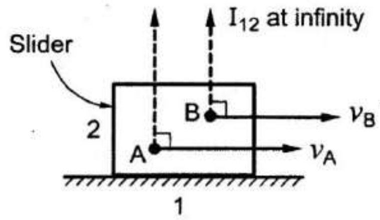
where n = Number of links

1. When two links are connected by a pin joint, the instantaneous centre lies on the centre of the pin, as shown in Figs. (*a*) and (*b*).



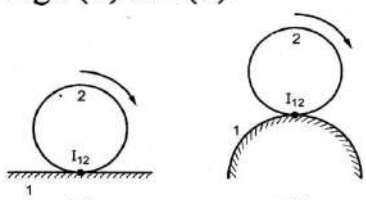
(*a*) (*b*)

2. When the two links have a sliding contact, the instantaneous centre lies at infinity in a direction perpendicular to the path of motion of the slider, as shown in Fig. (*c*).



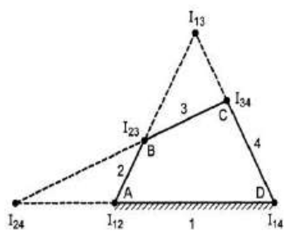
(*c*)

3. When the two links have a pure rolling contact, the instantaneous centre lies on their point of contact, as shown in Figs. (*d*) and (*e*).



(*d*) (*e*)

Types of Instantaneous Centres



Fixed Instantaneous Centers → The centers I_{12} and I_{14} are remain same place for all configuration of mechanism

Permanent Instantaneous Centers → The centers I_{23} and I_{34} are move when the mechanism move

Neither Fixed nor Permanent Instantaneous Centers → The centers I_{13} and I_{24} are vary with the configuration of the mechanism move

Kennedy's theorem: *"If three bodies have relative motion with each other, then their relative instantaneous centres must lie on a straight line."*

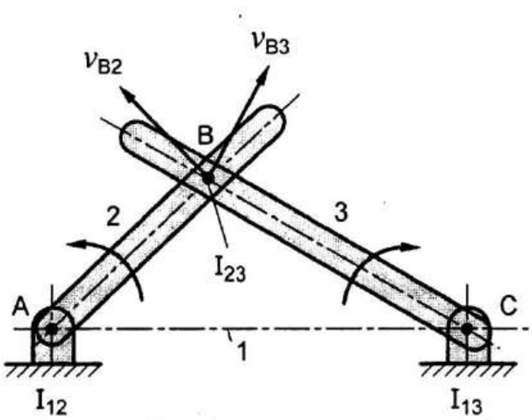


Illustration: Fig. shows any three kinematic links having motion in one plane. The number of instantaneous centres for three links,

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3 \quad [\because n = 3]$$

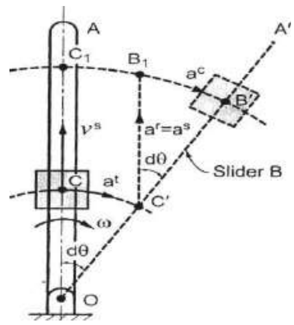
Two instantaneous centres I_{12} and I_{13} are permanent instantaneous centres at the pin joints A and C respectively. According to Kennedy's theorem, the third instantaneous centres I_{23} must lie on the line joining I_{12} and I_{13} (not at the point A).

Coriolis component of Acceleration

when the distance between the two points is not fixed (*i.e.,* the distance varies), then the total acceleration will have one more additional component known as Coriolis component of acceleration. It is represented by a^c_{AB}. In such cases, *the total acceleration of that link is the vector sum of its radial, tangential and Coriolis acceleration components i.e.,* $a_{AB} = a^r_{AB} + a^t_{AB} + a^c_{AB}$.

Magnitude of coriolis component of acceleration

Consider a link OA which has a slider B which is free to slide, as shown in Fig. With O as centre, let the link OA move, with a uniform angular velocity ω, to its new position OA′ such that it is displaced $d\theta$ in time dt. The slider B moves outwards with sliding velocity v^s on link OA and occupies the position B′ in the same interval of time.



Motions

(*i*) Motion from C to C′ due to rotation of link OA. It is caused by tangential component of acceleration a^t.

(*ii*) Motion from C′ to B_1 due to outward motion along the link OA. It is caused by radial component a^r

(*iii*) Motion from B_1 to B′ is caused by Coriolis component of acceleration a^c.

From the geometry of Fig.

$$\begin{aligned}
\text{Arc } B_1B' &= \text{Arc } C_1B' - \text{Arc } C_1B_1 \\
&= \text{Arc } C_1B' - \text{Arc } CC' \\
&= OC_1\,(d\theta) - OC\,(d\theta) = (OC_1 - OC)\,d\theta \\
&= CC_1 \times d\theta = C'B_1 \times d\theta
\end{aligned}$$

We know that $\quad C'B_1 = \text{Motion of the slider} \times \text{Time} = v \times dt$

and $\quad \dfrac{d\theta}{dt} = \omega \;\text{ or }\; d\theta = \omega \cdot dt$

Now equation (i) can be written as

$$\text{Arc } B_1B' = v^s \cdot d\theta \cdot \omega \cdot dt = v^s \omega (dt)^2 \quad \ldots (ii)$$

Considering the acceleration of the slider (a^c) as constant, we get

$$\text{Arc } B_1B' = \frac{1}{2} a^c (dt)^2 \qquad [\because s = ut + \frac{1}{2} at^2 \text{ and } u = 0]$$

For small value of $d\theta$,

$$\text{Arc } B_1B' = B_1B'$$

$$\therefore \quad B_1B' = \frac{1}{2} a^c (dt)^2 \quad \ldots (iii)$$

From equations (ii) and (iii), we get

$$v^s \cdot \omega (dt)^2 = \frac{1}{2} a^c (dt)^2$$

or Coriolis component of acceleration, $\boxed{a^c = 2 \cdot v^s \cdot \omega}$

where v^s = Velocity of sliding, and

ω = Angular velocity of link OA.

Kinematic Synthesis

Synthesis of mechanisms involve the design of various parts of machine concerning

(i) its shape and size,

(ii) materials to be used,

(iii) the arrangement of parts

Examples; To design a cam to give a follower given displacements and to determine number of teeth on the members in a gear train in order to produce a desired velocity ratio, are some examples of synthesis of mechanisms.

Type of Synthesis

Type synthesis refers to selection of type of mechanisms (such as gears, cams, belts, etc.) to be employed for a given application.

This is the first stage in the kinematic synthesis. During this stage, the design aspects such as space consideration, safety aspects, economics, manufacturing processes, materials selection, *etc.*, have to be considered.

Number Synthesis

Number synthesis refers to the determination of the number and order of links and joints required for a specified motion.

This is the second stage of kinematic synthesis. It involves the movability/mobility studies of a mechanism by the use of Grübler's criterion.

Dimensional Synthesis

Dimensional synthesis refers to the determination of the dimensions of parts (i.e., lengths and angles) so as to accomplish specified task and desired motion characteristics.

This is the third stage in kinematic synthesis.

Types of synthesis Problems (Tasks of kinematic synthesis)

1. Function Generation

- ✓ *In function generation, the motion of input link is correlated to the motion of output link.*
- ✓ Frequent requirement in design is that of causing an output member to rotate, or oscillate according to a specified function of time or function of input motion. This is called function generation.

2. Path Generation

- ✓ *In path generation, a point on the coupler link is constrained to describe a path with reference to a fixed frame.*
- ✓ The path is generally an arc of a circle, ellipse or a straight line. The coupler is a floating link.

3. Motion Generation (Rigid Body Guidance)

- ✓ *In motion generation, a mechanism is designed to guide a rigid body in a prescribed path.*
- ✓ This rigid body is considered to be the coupler or the floating link of a mechanism.

Problems

In a four bar chain ABCD,AD is fixed and is 120mm long .the crank AB is 30mm long and rotates at 100r.p.m clockwise. While the link CD=60mm oscillates about D,BC and AD are of equal length. Find the angular velocity and angular acceleration of the link BC when BAD= 60degree.

☺ ***Solution :*** ***(Relative velocity method)***

Given Data :

$$N_{BA} = 100 \text{ r.p.m.}$$

$$\omega_{BA} = \frac{2\pi N_{BA}}{60} = \frac{2\pi \times 100}{60} = 10.47 \text{ rad/sec}$$

(a) Configuration diagram :

Scale : 20 mm = 1 cm

(b) Velocity diagram :

Scale : 0.0628 m/s = 1 cm

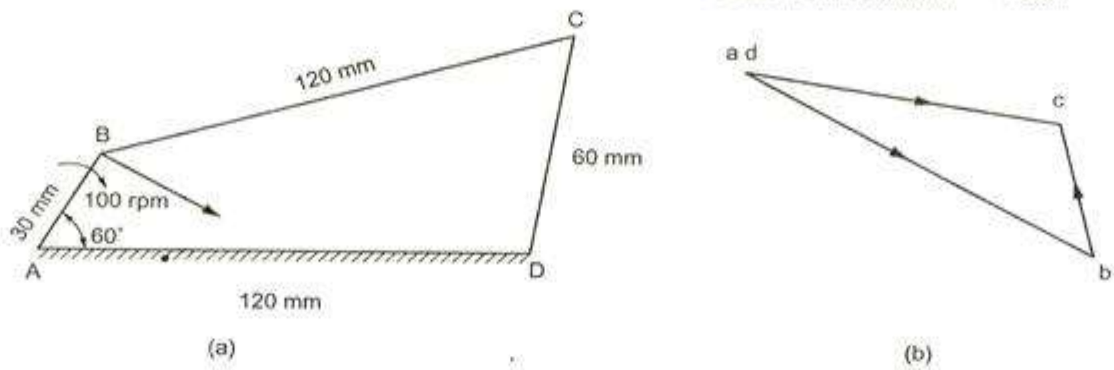
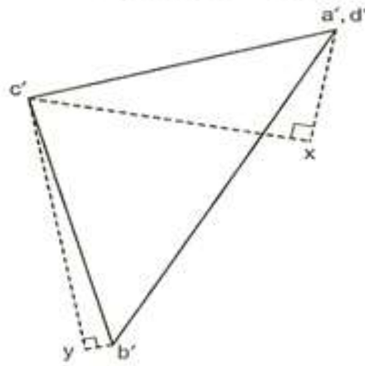


Fig. 2.31.

$$v_{BA} = \omega_{BA} \times AB = 10.47 \times 0.03$$

$$v_{BA} = 0.3141 \text{ m/s}$$

(c) Acceleration diagram : Scale : 0.6577 m/s² = 1 cm



1. From point '*a*' draw a vector line perpendicular to the velocity vector '*ab*' with magnitude equal to $\omega_{AB}^2 \times AB$, which will indicate radial component of acceleration of link AB.
2. Since the angular acceleration of the link AB is not given the tangential component of acceleration of link AB is zero. Hence radial component equal to acceleration of link AB.
3. From point '*d*' draw a line perpendicular to vector '*cd*' with magnitude equals to radial component of acceleration of link 'CD'.

   - $$a^r_{CD} = \frac{v^2_{CD}}{CD} = \frac{0.2387^2}{0.06} = 0.9496 \text{ m/s}^2$$

   and locate the point as '*x*'.
4. From *x* draw a line perpendicular to radial component of link CD.
5. From point '*b*' draw a line perpendicular to vector '*cb*' with magnitude equals to radial component of acceleration of the link 'CB'.

   $$a^r_{CB} = \frac{v^2_{CB}}{CB} = \frac{0.1256^2}{0.12} = 0.1315 \text{ m/s}^2$$ and locate the point as '*y*'.
6. From point '*y*' draw a line perpendicular to vector '*b'y'*' and locate the line joining point as '*c*'.

Results :

1. ***Angular velocity of the link BC :***

$$\omega_{BC} = \frac{v_{BC}}{BC} = \frac{0.1256}{0.12} = \mathbf{1.0467 \text{ rad/sec}} \quad \textbf{Ans.}$$

2. ***Angular acceleration of the link BC :***

$$\alpha_{BC} = \frac{a^t_{BC}}{BC}$$

$$a^t_{BC} = 3.9 \times 0.6577 = 2.565 \text{ m/sec}^2$$

[∵ 3.9 = from acc. diagram ; 0.6577 = scale of the acc. diagram]

$$\alpha_{BC} = \frac{2.565}{0.12} = \mathbf{21.375 \text{ rad/sec}^2} \quad \textbf{Ans.}$$

2. In a four link mechanism , the crank AB rotates at 36 rad/sec. the lengths of the links are AB=200mm,BC=400mm,CD=450mm and AD=600mm is the fixed link. At the instant when AB is at right angle to AD. Determine the velocity and acceleration at (i) the mid point of the link BC.

☺ ***Solution :***

Given Data : $\omega_{BA} = 36$ rad/sec ; and $\angle BAD = 90°$;

AB = 200 mm ; BC = 400 mm ;

CD = 450 mm ; AD = 600 mm ;

AD = Fixed.

(a) Configuration diagram : ***(b) Velocity diagram :***

Scale : 100 mm = 1 cm ***Scale*** : 1.44 m/s = 1 cm

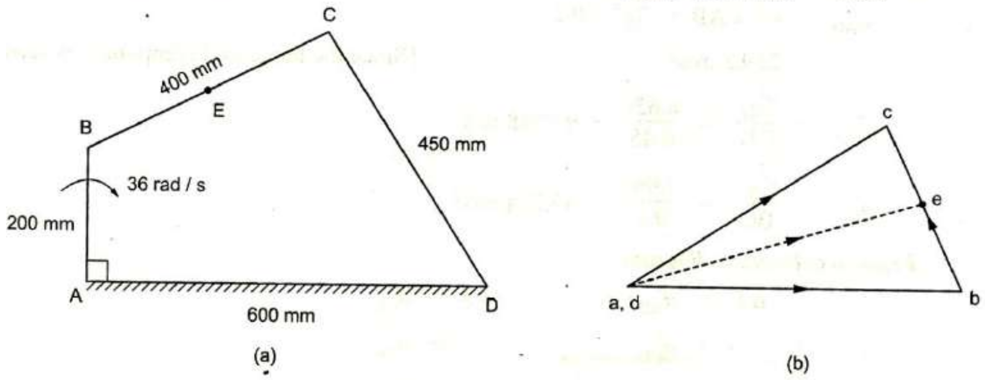


Fig. 2.33.

$v_{BA} = \omega_{BA} \times AB = 36 \times 0.2 = 7.2$ m/s

(c) Acceleration diagram : Scale : 64.8 m/s² = 1 cm

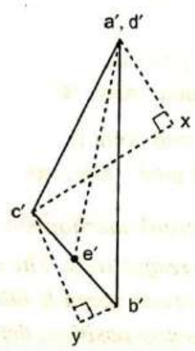


Fig. 2.33(c)

Procedure :

(a) Configuration diagram : Refer Fig.2.33(a).

The configuration diagram can be drawn as explained in the Example 2.2.

(b) Velocity diagram :

The velocity diagram can be drawn as explained in the Example 2.2.

1. Locate the point '*e*' at the centre of vector *bc* to find velocity of mid point of link BC
2. Join '*e*' and '*a*'. Now the vector v_{EA} will give the velocity of the mid point.

i.e., $v_{EA} = \overline{ea} = (4.55 \times 1.44) =$ **6.552 m/s Ans.** ☞

(c) Acceleration diagram :

Draw the acceleration diagram as explained in the previous problem.

$$a_{AB} = \omega^2 \times AB = 36^2 \times 0.2$$

$$= 259.2 \text{ m/s}^2$$ [Since the tangential component is zero]

$$a^r_{CD} = \frac{v^2_{CD}}{CD} = \frac{6.62^2}{0.45} = 97.388 \text{ m/s}^2$$

$$a^r_{BC} = \frac{v^2_{BC}}{BC} = \frac{3.96^2}{0.4} = 39.204 \text{ m/s}^2$$

From acceleration diagram :

$$a'b' = a_{AB} \qquad c'x = a^t_{CD}$$
$$a'x = a^r_{CD} \qquad c'y = a^t_{BC}$$
$$b'y = a^r_{BC} \qquad a^t_{AB} = 0$$
$$a_{AE} = a'e' = 3.4 \times 64.8 \qquad [\because a'e' = 3.4 \text{ cm} ;\ \text{scale} = 64.8 \text{ m/s}]$$
$$= \mathbf{220.32 \ m/s^2} \quad \textbf{Ans.}$$

Results :

1. Linear velocity at the mid point E,

$$v_E = \mathbf{6.552 \ m/s} \quad \textbf{Ans.}$$

2. Linear acceleration at the mid point E,

$$a_E = \mathbf{220.32 \ m/s^2} \quad \textbf{Ans.}$$

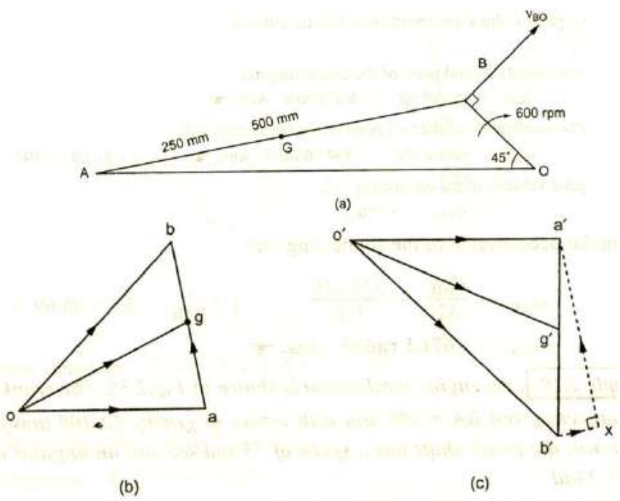
3. In a slider crank mechanism , the length of crank OB and connecting rod AB are 125mm and 500mm respectively. The center of gravity of the connecting rod is 250mm from the slider A. the crank speed is 600r.p.m clockwise . when the crank has turned 45 degree form the inner dead center position , determine
1. Linear velocity and acceleration of the mid point of the connecting rod and
2. Angular velocity and angular acceleration of the connecting rod.

☺ ***Solution : Given Data :*** Slider crank mechanism,

$$OB = 125 \text{ mm} ; \qquad AB = 500 \text{ mm} ;$$
$$AG = 250 \text{ mm} ; \qquad N_{BO} = 600 \text{ r.p.m. (cw)}$$
$$\omega_{BO} = \frac{2\pi \times 600}{60} = 62.83 \text{ r/s} ; \qquad \theta = 45° \text{ from the inner dead centre.}$$
$$v_{BO} = \omega_{BO} \times OB = 62.83 \times 0.125 = 7.854 \text{ m/s}$$

(a) Configuration diagram :
Scale : 62.5 mm = 1 cm

(b) Velocity diagram :
Scale : 1.96 m/s = 1 cm



(c) Acceleration diagram : Scale : 98.69 m/s² = 1 cm

4. In the mechanism as shown in fig, the crank OA rotates at 20r.p.m anticlockwise and gives motion to the sliding B and D. the dimensions of various links are OA=300mm; AB= 1200mm; BC=450mm and CD= 450mm.

For the given configuration , determine

(i) velocity of sliding at B and D,

(ii) angular velocity of CD,
(iii) linear acceleration of D and
(iv) angular acceleration of CD.

☺ ***Solution : Given Data :*** $N_{OA} = 20$ r.p.m. anticlockwise ;

$OA = 300$ mm ;

$AB = 1200$ mm ;

$BC = 450$ mm ;

$CD = 450$ mm ;

$$\omega_{OA} = \frac{2\pi \times 20}{60}$$

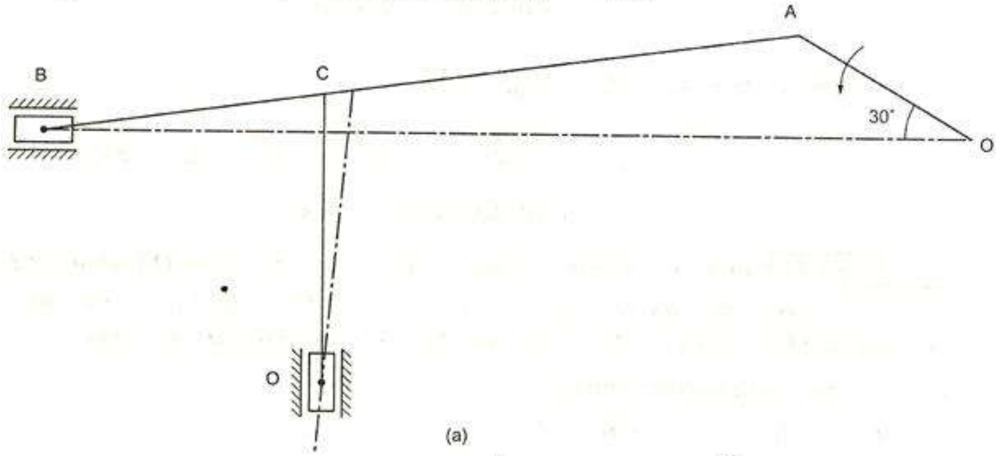
$$= 2.094 \text{ rad/sec ;}$$

$$v_{OA} = 0.6283 \text{ m/s.}$$

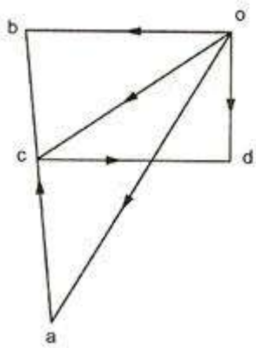
(a) Configuration diagram :
Scale : 100 mm = 1 cm

(b) Velocity diagram :
Scale : 0.1256 m/s = 1 cm

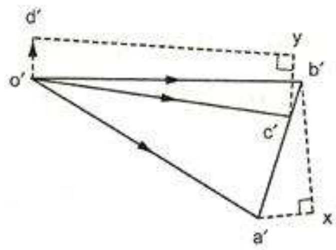
(c) Acceleration diagram : Scale : 0.326 m/s^2 = 1 cm



(a)



(b)



(c)

(c) Acceleration diagram :

1. Draw vector $o'a'$ parallel to OA or perpendicular to vector oa to some suitable scale, to represent the radial component of the acceleration of A with respect to 'o' or simply the acceleration of A, such that vector $o'a' = a^r_{AO} = a_A = 1.323$ m/s^2

[$\because a_A = a^r_{AO}$; since there is no tangential component]

2. From point a', draw vector $a'x$ parallel to AB to represent the radial component of the acceleration of B with respect to A, such that

$$\text{vector } a'x = a^r_{BA} = \frac{v^2_{AB}}{AB} = \frac{(4.3 \times 0.1256)^2}{1.2} \quad ; \quad a'x = 0.243 \text{ m/s}^2$$

3. From point x, draw vector xb' perpendicular to AB or parallel to vector ab to represent the tangential component of the acceleration of B with respect to A (*i.e.*, a^t_{BA}) whose magnitude is not yet known.

4. From point o', draw vector $o'b'$ parallel to the path of motion of B (which is along BO) to represent the acceleration of B (a_B). The vectors xb' and $a'b'$ intersect at b', join $a'b'$. The vector $a'b'$ represents the acceleration of B with respect to A.

5. Divide vector $a'b'$ at c' in the same ratio as c divides AB in the space diagram. In other words

$$\frac{BC}{CA} = \frac{b'c'}{c'a'} \text{ (or)} \qquad a'c' = \frac{AC}{AB} \times a'b' = \frac{750}{1200} \times 2.2$$

$$a'c' = 1.375 \text{ cm}$$

$$a'c' = 1.375 \times 0.329 = 0.452 \text{ m/s}^2$$

6. From point c', draw vector $c'y$ parallel to CD to represent the radial component of the acceleration of D with respect to c, such that

$$\text{vector } c'y = a^r_{DC} = \frac{v^2_{DC}}{DC} = \frac{(3 \times 0.1256)^2}{0.45}$$

$$c'y = 0.315 \text{ m/s}^2$$

7. From point y, draw vector yd' perpendicular to CD to represent the tangential component of acceleration of D with respect to C whose magnitude is unknown.

8. From point o', draw vector $o'd'$ parallel to the path of motion of D (which is along the vertical direction) to represent the acceleration of D. The vectors yd' and $o'd'$ intersect at d'.

By measurement, we find that the

$$\text{linear acceleration of D} = a_D = \text{vector } o'd' = 0.5 \times 0.329$$

$$a_D = 0.1645 \text{ m/s}^2$$

Results :

1. (a) Velocity of slider at B = v_B = vector ob = **0.401 m/s Ans.** ☜

   (b) Velocity of slider at D = v_D = vector od = **0.238 m/s Ans.** ☜

2. Angular velocity of CD = ω_{OA} = **0.809 rad/sec Ans.** ☜

3. Linear acceleration of D = a_D = **0.1645 m/s² Ans.** ☜

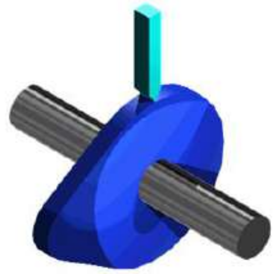
4. Angular acceleration of CD $= \alpha_{CD} = \dfrac{a^t_{CD}}{CD} = \dfrac{\text{vector } yd'}{CD}$

$$\alpha_{CD} = \frac{3.9 \times 0.329}{0.45} = \frac{1.2831}{0.45} = \mathbf{2.85\ rad/sec^2} \quad \textbf{Ans.}$$ ☜

Unit 3- KINEMATICS OF CAM MECHANISMS

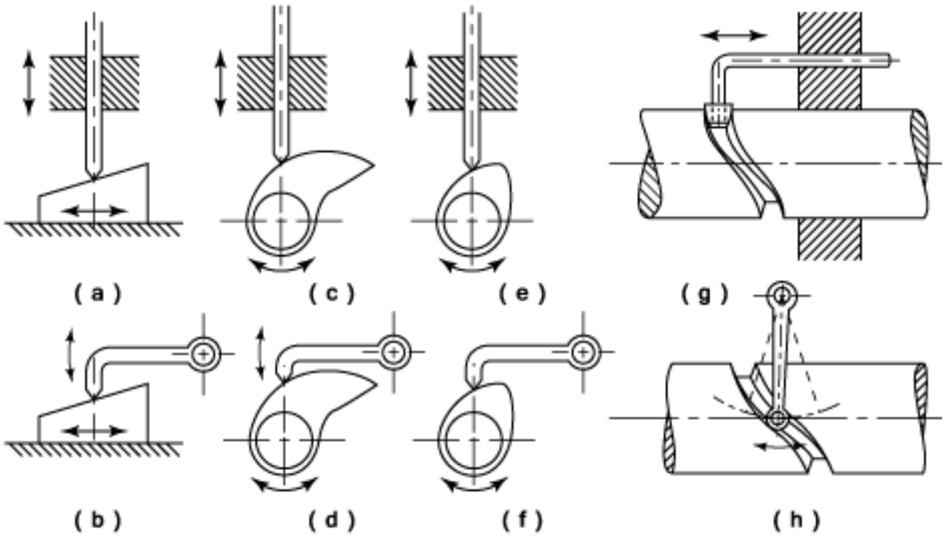
Cam

A ***cam*** may be defined as a machine element having a curved outline or a curved groove, which, by its oscillation or rotation motion, gives a predetermined specified motion to another element called the ***follower*** . The cam has a very important function in the operation of many classes of machines, especially those of the automatic type, such as printing presses, shoe machinery, textile machinery, gear-cutting machines, and screw machines.



Types of Cam

A Cam **is a reciprocating, oscillating or rotating body which imparts reciprocating or oscillating motion to a second body, called the follower, with which it is in contact. The shape of the cam depends upon its own motion, the required motion of the follower and the shape of the contact face of the follower.**



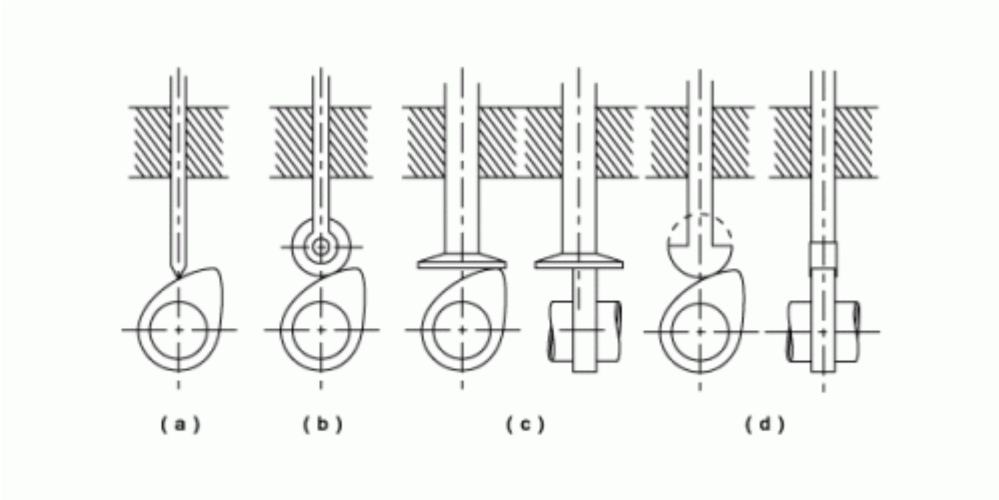
Cams are classified according to the direction of displacement of the follower with respect to the axis or oscillation of the cam. The two most important types are :

- **Disc or Radial Cams** In these the working surface of the cam is shaped that the reciprocation or oscillation of the follower is in a plane at right angles to the axis of the cam. (see examples c; d; e; f above)
- **Cylindrical Cams** These are often used in machine- tools and the cam imparts an oscillation or reciprocation to the follower in a plane parallel to the axis of the cam. (see examples g and h above)

Types Of Follower.

Followers can be divided according to the shape of that part which is in contact with the cam.. The following diagram shows some of the more common types:

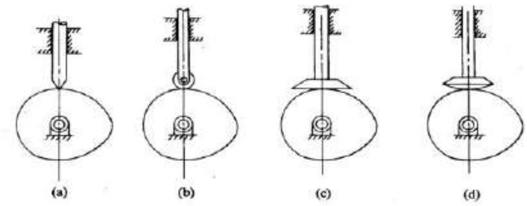
a. **Knife edged**. These are not often used due to the rapid rate of wear of the knife edge. This design produces a considerable side thrust between the follower and the guide.

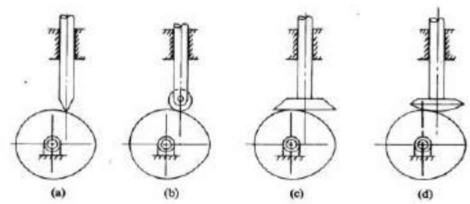


b. **Roller Follower**. The roller follower has the advantage that the sliding motion between cam and follower is largely replaced by a rolling motion. Note that sliding is not entirely eliminated since the inertia of the roller prevents it from responding instantaneously to the change of angular velocity required by the varying peripheral speed of the cam. This type of follower also produces a considerable side thrust.
c. **Flat or Mushroom Follower**. These have the advantage that the only side thrust is that due to friction between the contact surfaces of cam and follower. The relative motion is one of sliding but it may be possible to reduce this by off setting the axis of the follower as shown in the diagram. This results in the the follower revolving under the influence of the cam.
d. **Flat faced Follower**. These are really an example of the mushroom follower and are used where space is limited. The most obvious example being automobile engines.

Radial Followers

When the motion of the follower is along an axis passing through the centre of the cam, it is known as radial followers. Above figures are examples of this type.

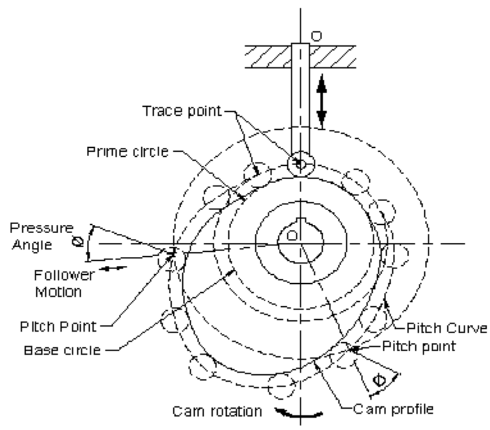




When the motion of the follower is along an axis away from the axis of the cam centre, it is called off-set follower. Above figures are examples of this type.

Cam Nomenclature

- **Cam profile:** The outer surface of the disc cam.
- **Base circle :** The circle with the shortest radius from the cam center to any part of the cam profile.
- **Trace point:** It is a point on the follower, and its motion describes the movement of the follower. It is used to generate the pitch curve.



- **Pitch curve :** The path generated by the trace point as the follower is rotated about a stationery cam.

- **Prime circle:** The smallest circle from the cam center through the pitch curve
- **Pressure angle:** The angle between the direction of the follower movement and the normal to the pitch curve.
- **Pitch point:** Pitch point corresponds to the point of maximum pressure angle.
- **Pitch circle:** A circle drawn from the cam center and passes through the pitch point is called Pitch circle
- **Stroke:** The greatest distance or angle through which the follower moves or rotates

Types of follower motion

1. Uniform motion (constant velocity)
2. Simple harmonic motion
3. Uniform acceleration and retardation motion
4. Cycloidal motion

Problem

1. A cam operating a knife edged follower has the following data:

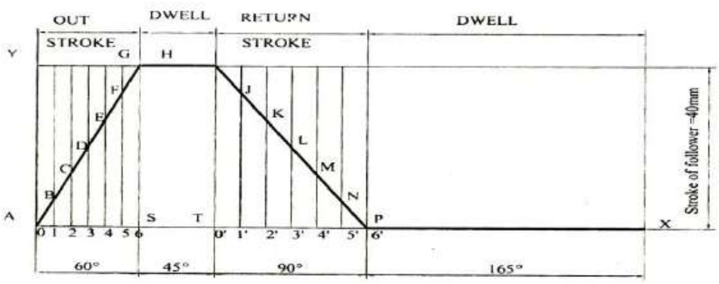
(a) Follower moves outward through 40 mm during 60* of cam rotation

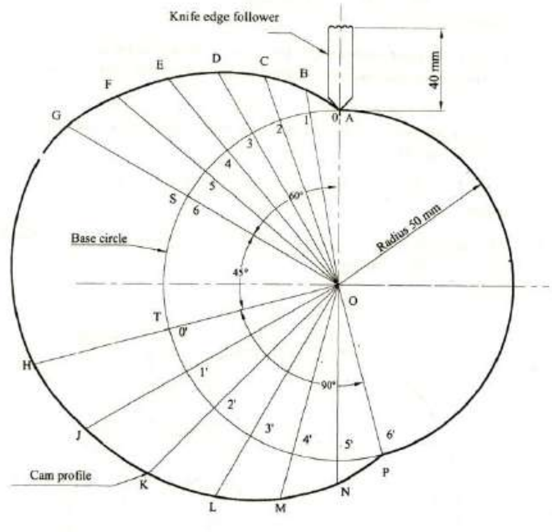
(b) Follower dwells for the next 45*.

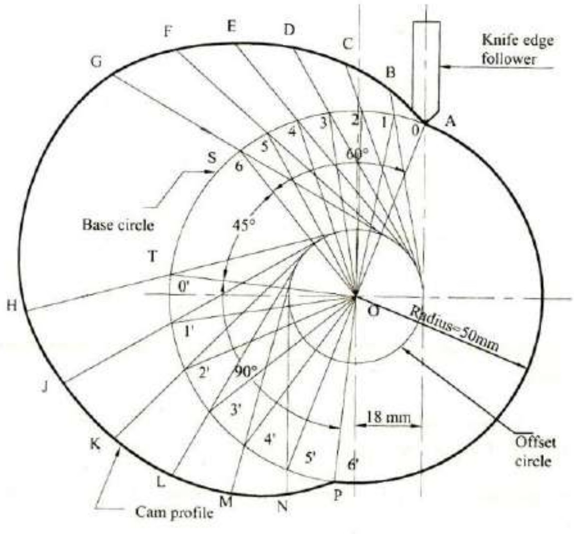
(c) Follower returns to its original position during next 90*.

(d) Follower dwells for the rest of the rotation.

The displacement of the follower is to take place with uniform velocity during the both outward and return strokes. The least radius of the cam is 50 mm. Draw the profile of the cam when (1) the axis of the follower passes through the cam axis, and (2) the axis of the Follower is offset by 18 mm towards right from the cam axis.







2. **A cam rotating clockwise with a uniform speed is to give the roller follower of 20 mm diameter with the following motion.**

   (a) Follower to move outward through a distance of 30 mm during 120* of cam rotation.

   (b) Follower to dwell for 60* of cam rotation.

   (c) Follower to return its initial position during 90* of cam rotation and

   (d) Follower to dwell for the remaining 90* of cam rotation.

The minimum radius of the cam is 45 mm and the displacement of the follower is to take place with SHM on both the outward and return strokes. Draw the cam profile when

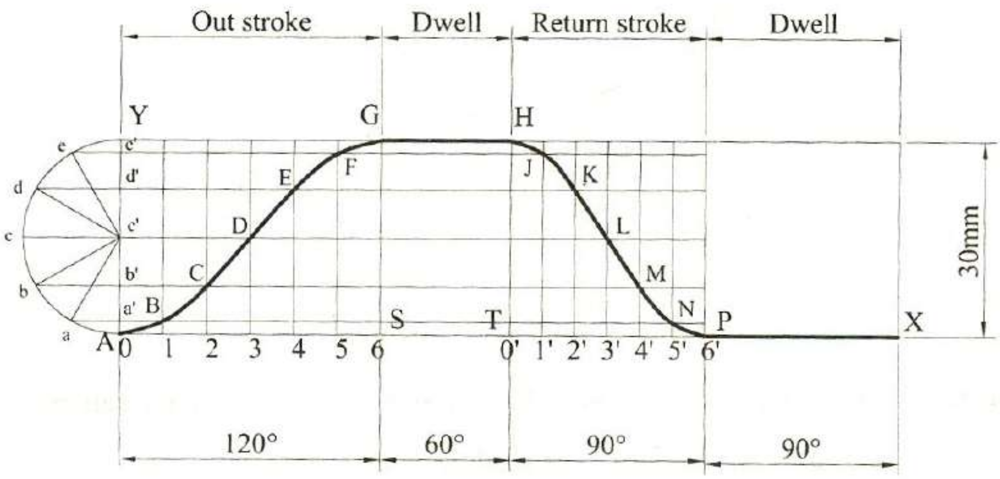
   (1) The line of stroke of the follower passes through the axis of the cam shaft, and

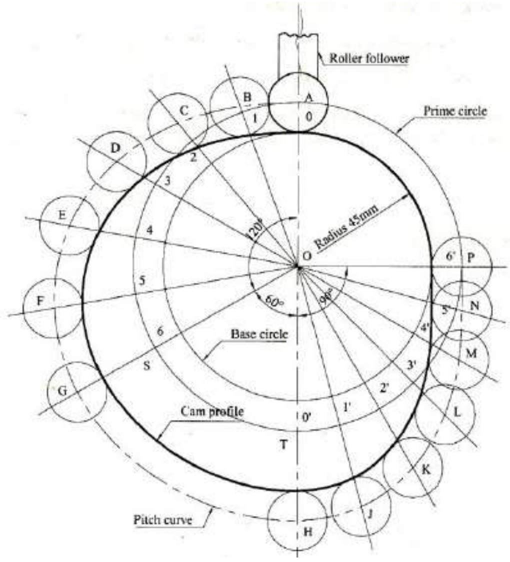
   (2) The line of stroke of the follower is offset by 15 mm from the axis of the cam.

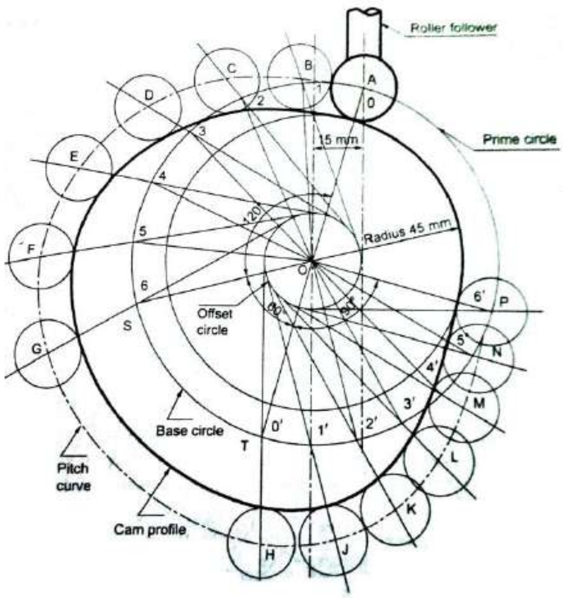
☺ ***Solution : Given Data :***

$$S = 30 \text{ mm} = 0.03 \text{ m};$$

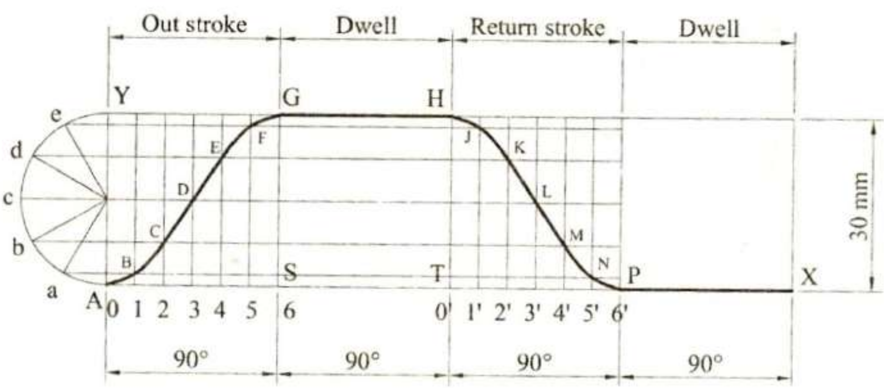
Minimum radius of cam = 45 mm

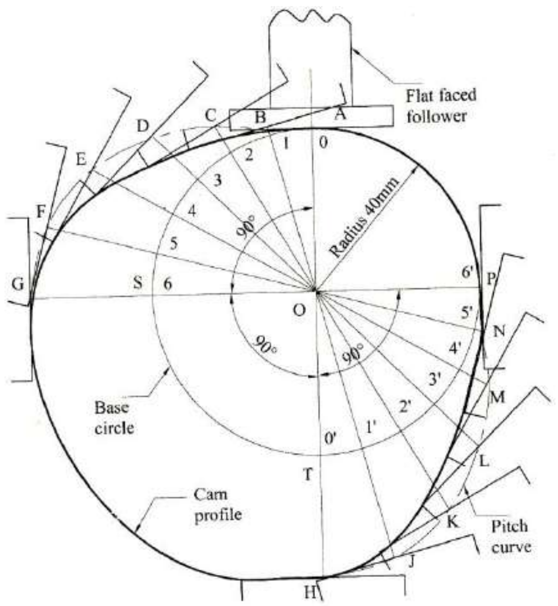






3. A cam drives a flat reciprocating follower in the following manner. During first 90* rotation of cam, follower moves outwards through a distance of 30 mm with SHM. The follower dwells during next 90* of cam rotation. During next 90* of cam rotation, the follower moves inwards with SHM. The follower dwells for the next 90* of cam rotation. The minimum radius of the cam is 40 mm. Draw the profile of the cam. Also calculate the maximum values of velocity and acceleration when the cam rotates at 10 rad/s.





Maximum velocity of follower during ascent and descent :

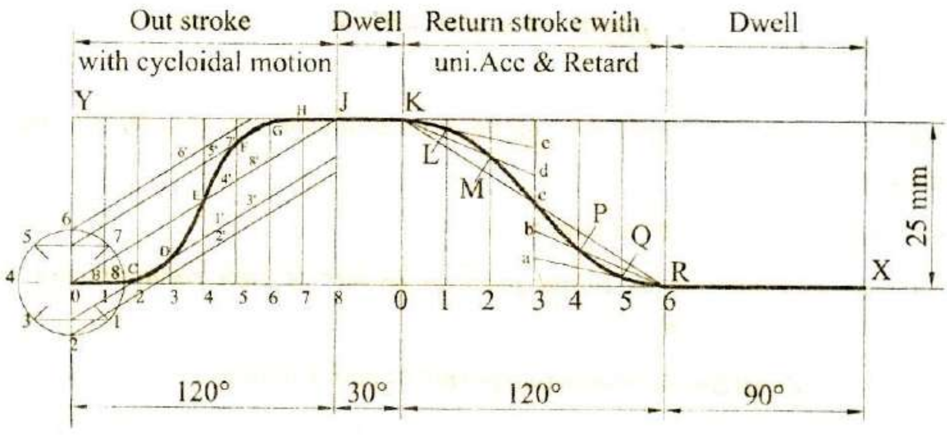
$$v_0 = v_R = \frac{\pi \omega S}{2 \theta_0} = \frac{\pi \omega S}{2 \theta_R} \qquad [\because \theta_0 = \theta_R = 90°]$$

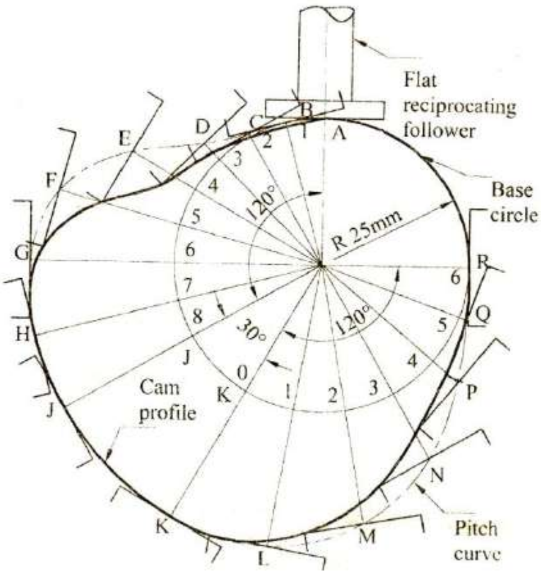
$$v_0 = v_R = \frac{\pi \times 10 \times 0.03}{2 \times 1.571} = \mathbf{0.3\ m/sec}\ \ \textbf{Ans.}$$

Maximum acceleration of follower during ascent and descent :

$$a_0 = a_R = \frac{\pi^2 \omega^2 S}{2 (\theta_0)^2} = \frac{\pi^2 \omega^2 S}{2 (\theta_R)^2} = \mathbf{6\ m/s^2}\ \ \textbf{Ans.}$$

4. A flat faced mushroom follower is operated by a uniformly rotating cam. The follower is raised through a distance of 25 mm in 120* rotation of the cam, remains at rest of for the next 30* and is lowered during further 120* rotation of the cam. The raising of the follower takes place with cycloidal motion and lowering with uniform acceleration and deceleration. The least radius of the cam is 25 mm which rotates at 300 rpm.
Draw the cam profile and determine the values of the maximum velocity and maximum acceleration during rising and maximum velocity and uniform acceleration and deceleration during lowering of the follower.

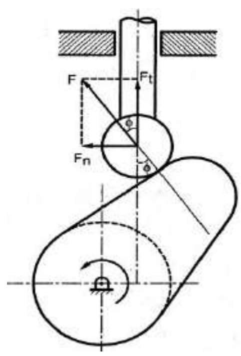




Tangent Cam

When the flanks of the cam are straight and tangential to the base circle and nose circle, then the cam is known as tangent cam.

Pressure Angle



Definition: the pressure angle (ϕ) is the angle between the line of action of follower and corresponding normal to the pitch curve through the trace point.

The pressure angle is very important in cam design as it measures the effectiveness of cam to transfer driving force to the follower.

The greater the pressure angle, the higher will be the side thrust. Thus the side thrust of the follower can be reduced by decreasing the pressure angle.

Methods to reduce pressure angle: The pressure angle can be reduced by using any one or combination of the following ways:

(i) By increasing the cam size (*i.e.,* by increasing the prime circle radius),

(ii) By adjusting the offset of the follower,

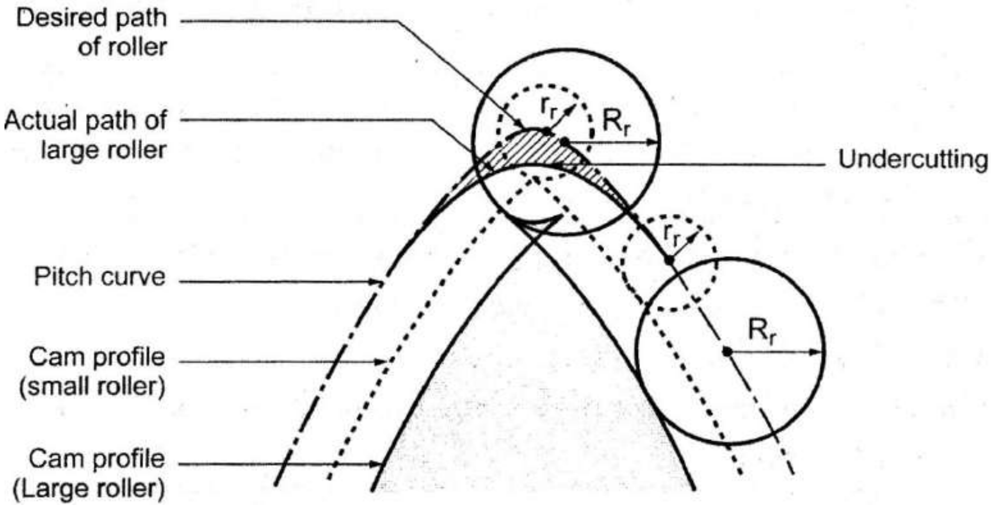
(iii) By changing the follower motion type,

(iv) By reducing the follower total rise, and

(v) By increasing the amount of cam rotation for a given follower displacement.

Under Cutting in Cam

If the curvature of the pitch curve is too sharp, then the part of the cam shape would be lost and thereafter the intended cam motion would not be achieved. Such a cam is said to be undercut.



Methods to Avoid Undercutting in Cam

1. By decreasing the desired follower lift,
2. By increasing the cam rotation angle, and
3. By increasing the cam size (*i.e.*, the prime circle radius of the cam).

Unit 4 – GEARS AND GEAR TRAINS

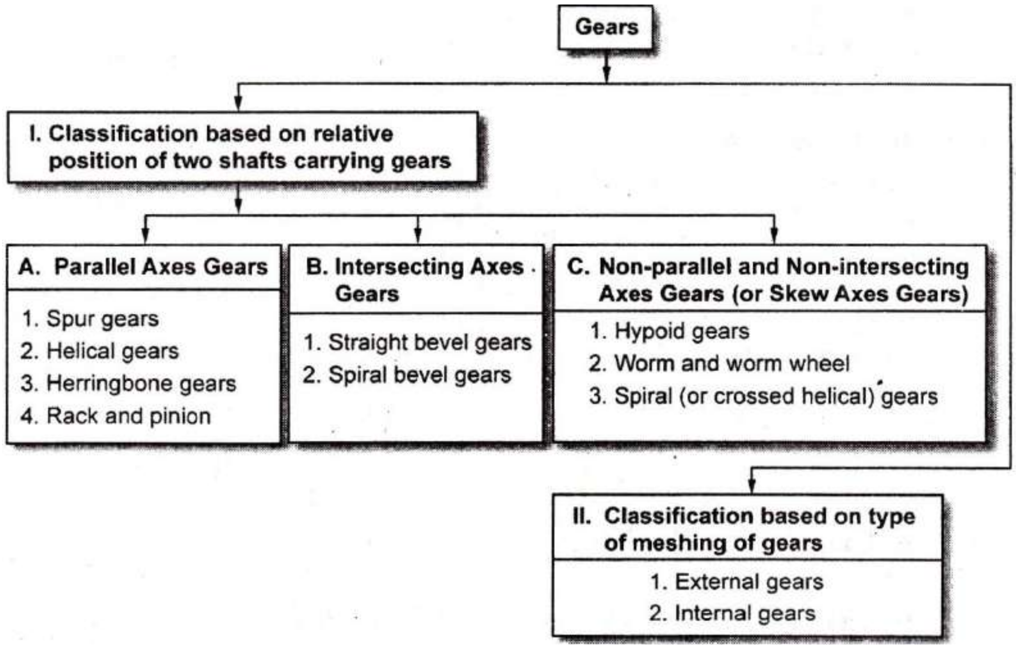
Gear

- Gears are toothed wheels used for transmitting motion and power from one shaft to another when they are not too far apart and when a constant velocity ratio is desired

Advantages of Gear Drive

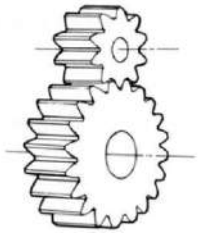
- Since there is no slip, so exact velocity ratio is obtained.
- Larger power can be transmitted.(Compare to Belt & Chain)
- Its more efficient (upto 99%)
- Its require less space

Types of Gears



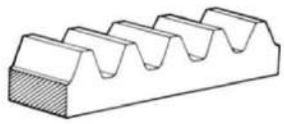
(1) Parallel Axes Gears

1 Spur Gear



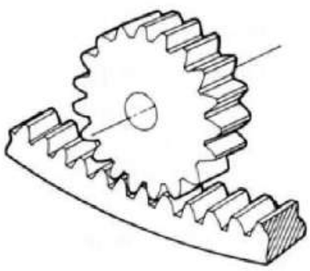
This is a cylindrical shaped gear, in which the teeth are parallel to the axis. It is the most commonly used gear with a wide range of applications and is the easiest to manufacture.

2 Gear Rack



This is a linear shaped gear which can mesh with a spur gear with any number of teeth. The gear rack is a portion of a spur gear with an infinite radius.

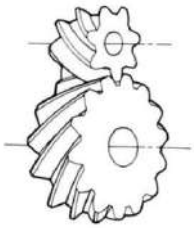
3 Internal Gear



Internal Gear and Spur Gear

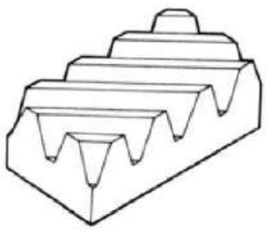
This is a cylindrical shaped gear, but with the teeth inside the circular ring. It can mesh with a spur gear. Internal gears are often used in planetary gear systems.

4 Helical Gear



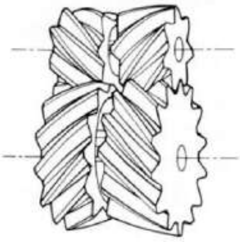
This is a cylindrical shaped gear with helicoid teeth. Helical gears can bear more load than spur gears, and work more quietly. They are widely used in industry. A disadvantage is the axial thrust force caused by the helix form.

5 Helical Rack



This is a linear shaped gear that meshes with a helical gear. A Helical Rack can be regarded as a portion of a helical gear with infinite radius.

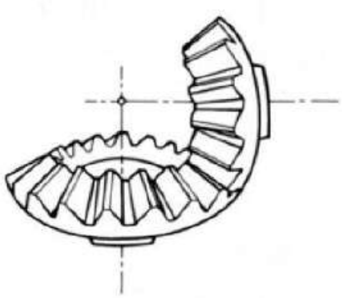
6 Double Helical Gear



A gear with both left-hand and right-hand helical teeth. The double helical form balances the inherent thrust forces.

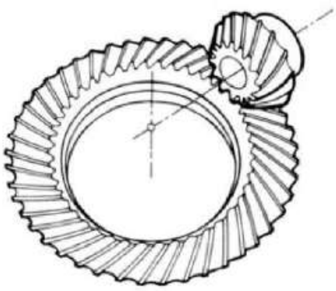
(2) Intersecting Axes

1 Straight Bevel Gear



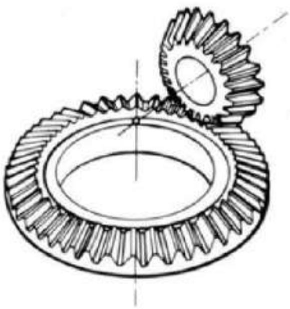
This is a gear in which the teeth have tapered conical elements that have the same direction as the pitch cone base line (generatrix). The straight bevel gear is both the simplest to produce and the most widely applied in the bevel gear family.

2 Spiral Bevel Gear



This is a bevel gear with a helical angle of spiral teeth. It is much more complex to manufacture, but offers higher strength and less noise.

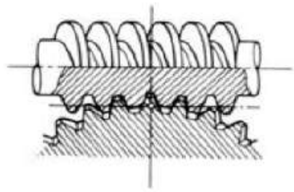
3 Zerol Bevel Gear



This is a special type of spiral bevel gear, where the spiral angle is zero degree. It has the characteristics of both the straight and spiral bevel gears. The forces acting upon the tooth are the same as for a straight bevel gear.

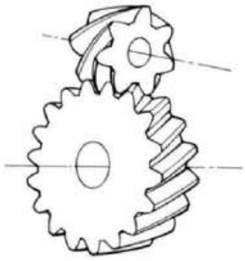
(3) Nonparallel and Nonintersecting Axes Gears

1 Worm Gear Pair



Worm gear pair is the name for a meshed worm and worm wheel. An outstanding feature is that it offers a very large gear ratio in a single mesh. It also provides quiet and smooth action. However, transmission efficiency is poor.

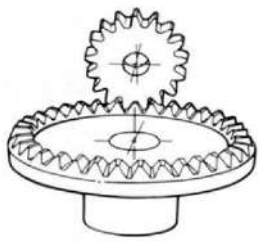
2 Screw Gear (Crossed Helical Gear)



A pair of cylindrical gears used to drive non-parallel and nonintersecting shafts where the teeth of one or both members of the pair are of screw form. Screw gears are used in the combination of screw gear / screw gear, or screw gear / spur gear. Screw gears assure smooth, quiet operation. However, they are not suitable for transmission of high horsepower.

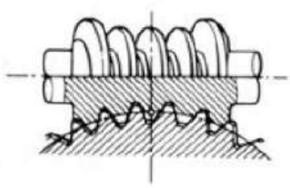
(4) Other Special Gears

1 Face Gear



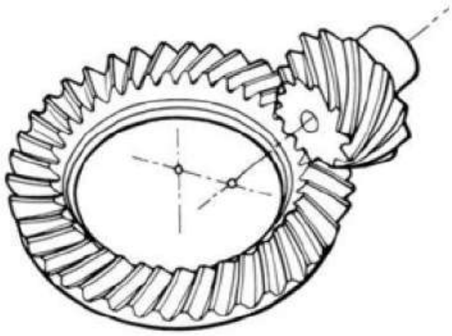
A pseudo bevel gear that is limited to 90° intersecting axes. The face gear is a circular disc with a ring of teeth cut in its side face; hence the name Face Gear.

2 Enveloping Gear Pair



This worm set uses a special worm shape that partially envelops the worm gear as viewed in the direction of the worm gear axis. Its big advantage over the standard worm is much higher load capacity. However, the worm gear is very complicated to design and produce.

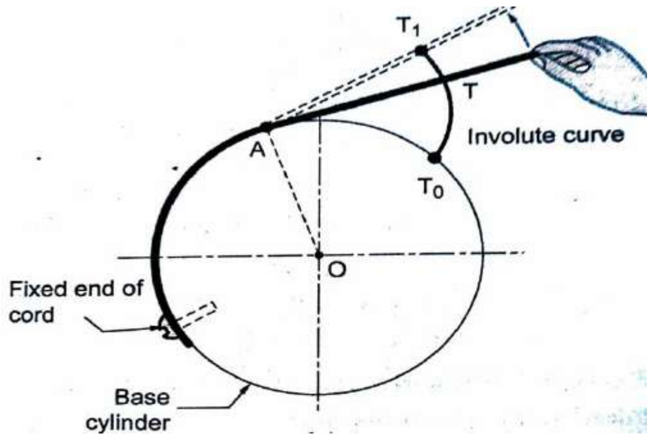
3 Hypoid Gear



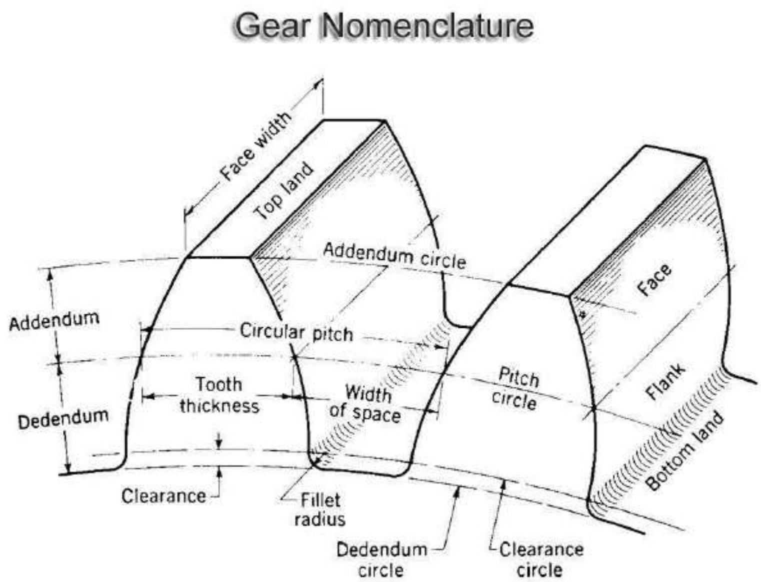
This gear is a slight deviation from a bevel gear that originated as a special development for the automobile industry. This permitted the drive to the rear axle to be nonintersecting, and thus allowed the auto body to be lowered. It looks very much like the spiral bevel gear. However, it is complicated to design and is the most difficult to produce on a bevel gear generator.

Involute tooth profiles

Involute curve is a path generated by a tracing point on a cord as the cord is unwrapped from a cylinder called the base cylinder



Spur Gear Terminology



Pinion

- The smallest of two matting gear. The largest often called the gears or the wheel.

pitch circle

- It is an imaginary circle, which is pure rolling action. For actual gear to give the same motion.

Pitch circle diameter:

It is diameter of the Pitch Circle. When the size of the gear is specified by pitch circle diameter. It is also known as pitch diameter.

Pitch point:

It is a common point of contact between the two pitch circle.

Pitch surface:

It is a surface of Rolling disc which meshing gears are replaced at pitch circle.

Pitch:

Two mating gear of pitch are same.That should be having Three Types of pitch.

Circular pitch

What is distance between circumference of a pitch circle from a point of one tooth to the corresponding point of adjacent tooth.

Diametral pitch

It is the ratio between the pitch circle diameter to number of teeth. it is called diametral pitch.

module pitch

It Is ratio between pitch circle diameter to the number of teeth.

Addendum Circle(tip circle):

The circle drawn through the top of the teeth and it is concentric with the Pitch Circle.

Addendum:

it is radial distance between the Pitch circle to bottom of the tooth.

Dedendum circle(root circle)

It is circle draw through bottom of the circle.

Dedendum:

The radial distance of teeth in between pitch circle to bottom of the teeth.

Clearance:

The Clarence is a radial distance between the top of teeth to the bottom of the teeth in two meshing gear. A circle is drawn through the top of the meshing gear is called as clearance circle.

Total depth:

It is radial distance between the addendum circle to dedendum circle of gear.
So we can calculate the total death:
Total depth = addendum + dedendum

Working depth:

It is a Radial distance between the addendum circle to clearance circlet.It is also equal to sum of addendum of the two meshing gear.

Tooth thickness:

The Measuring of width the tooth along the Pitch Circle.

Tooth space:

It is measurement by space of width between to adjacent teeth measuring along the pitch circle.

Backlash:

It is difference between space of the tooth and thickness of the tooth along the Pitch Circle.
Backlash = tooth space – tooth thickness

Face Width:

It is measurement of gear width by parallel to its axis.

Top land:

It is your surface of top of the teeth. it is called top land.

Bottom Land:

It is surface of the bottom of the teeth between adjacent fillets.

Face:

It is tooth surface between top Land to Pitch Circle.

Flank:

The tooth surface between bottom land to pitch circle including fillet.

Fillet:

It is curved shape of the toothed flank at root circle.
Pressure angle(angle of obliquity):
The angle between the common normal to two gearing of teeth at point of contact and a common tangent at a point of pitch Point. It is called pressure angle in the Gearing. The standard pressure angle of the teeth is 14 1/2 degree and 20 degree.

Path of contact:

The point of contact of two teeth from beginning to end of engagement.
Length of path of contact(contact length):
It is measured by length of the common normal cutoff by addendum circle of the pinion and wheel.

Arc of content:

The path is measured by a point on the circle from beginning to end of engagement in pair of teeth. The arc of contact is divided by two types. The are
(i) arc of approach
(ii) Arc of recess

Arc of approach: It is path of contact from the beginning of engagement to end of pitch point.
Arc of recess: it is path of contact from Pitch point to end of engagement of pair of teeth.

Velocity ratio:

It is ratio between speed of driving gear to speed of driven gears.

Contact ratio:

The ratio between length of Arc of contact to circular pitch.it is called as Contact ratio.

Law of Gearing:

To obtain a constant velocity ratio, the common normal of any instant of teeth at each point of contact should always pass through a pitch point, Situated on line joining center of rotation of the of mating gear.

Mean of Other Words:(The condition made by the tooth profile to maintain a constant angular velocity ratio in between two pair of gear. The fundamental of condition which must be satisfied designing the profile of the teeth of gear wheel.)

General form of gear tooth profile:

The shape of two curve that fulfill the law of gearing, it can be used in teeth profile. if the profile of teeth of one matting gear is arbitrarily and profile of other gear is determined,so to satisfy the law of gearing, it type of teeth is known as conjugate teeth. The gear are having conjugate teeth can be successfully used for power transmission of motion. but there are difficult to manufacturing by using a special device which are costly. So the conjugate teeth are not used in most common method.
So the common form of the profile used in actual practical purpose;
They are (i) involute profile (ii) cycloidal tooth profile

Standard System of gear tooth profile:

American gear manufacture Association(AGMA) and American National standard Institute(ANSI) for standardized following for form of gear teeth profile depending upon the pressure angle.

1. $14\frac{1}{2}^{0}$ composite system
2. $14\frac{1}{2}^{0}$ full depth involute system
3. 20^{0} full depth envelope system
4. 20^{0} degree stub involute system

Advantages of $14\frac{1}{2}^{0}$ involute system

It type of profile teeth could be having smooth and noiseless operation.
Very strong teeth
Advantages of 14 degree involute system
It will be reduced the risk of undercutting
It will be stronger teeth with a load carrying capacity
It will be having greater length of contact

Gear material

the modern industrial widely various gear materials are used. The gears material are classified in two types. They are Metallic and non metallic material.

(1) metallic material:

- **Steel**: it is more common material of manufacturing of gears. Almost all types of Steel have been used in this manufacturing purpose.
- To combine the property of toughness and teeth hardness, steel gears are heat treated.
- The Steel With Brinell hardness number < 350 are used to light and medium duty purpose. But >350 BHN are used to heavy duty and it require to compactness.
- The plain carbon steel used for medium duty application such as 50 C 8,45 C 8, 50 C 4 and 55 C 8.For the heavy duty application alloy Steel are used like as 40 Cr 1 , 30 Ni 3 Cr 65 Mo 55.For planetary gear train the recommended Steel of 35 Ni 1 Cr 60.

cast iron:

The cast iron is a extensive gear material. Because of low cost,good machinability, and moderate Mechanical properties. For general purpose, the large size gear made from grey cast iron (FG 200, FG 260 or FG 350)
Disadvantage: it is a low tensile strength

bronze:

- It is mainly used to worm gears drive because of ability of highest sliding load.
- It is most suitable for corrosion and wear problem.
  Disadvantage: there are costly material.
- The bronze Alloys like as manganese bronze, aluminium bronze, Silicon bronze or
- phosphorus bronze are used in manufacture of gears.

(2) non metallic material:

The non metallic materials like as wood, rawhide, synthetic resin, compressed paper are used for gears.
Advantage:i. Cost very low ii. Noiseless operation iii. Damping of shock and vibration.
Disadvantage: low heat conductivity and low load carrying capacity

Selection of Gears Materials:

Selection of gears material depends on the various reason as follows,

- Service type
- The speed of peripheral
- Manufacturing method
- Degree of accuracy requirement
- Material cost
- Wear and shock resistance
- Weight and space limitation

- Impact load , maximum load and life of gear
- The safety and other consideration

Gear manufacturing method:

The manufacture of gear followed by various process and that can be classified,

Gear milling process:

- Gear milling is a process of cutting a material by feeding a work piece under rotating of multiple tooth cutter. When the cutting action for many teeth around the milling cutter and providing a fast method of machining. So the machined surface maybe followed by flat,angular, or curved.The machining of surface in above require shape of material.
- Spur gear, helical gear and straight bevel gear are milled by this method.
- The machining of surface finish having about 3.2µm.

Gear generating

The gears are formed together in serious of passes by generating tool shape like as mating of gears one to another.The hubs or shapers can be used by this method. The surface finish of machining process can be obtained by 1.6µm.

Gear hobbing:

it is an simply method, which cutting tool shaped in worm. The gear hobbing method can produce almost any Shape of external tooth form expect bevel gears. It also control tooth spacing, lead angle and profile.

Gear shaping:

The gear shaper is used and the teeth may be generated with either rack cutter or pinion cutter. It can produce internal and external spur gears, helical gears, herringbone gears and face gears.

Gear moulding:

It is achieved by mass production of gears.

Injection moulding:

It can produce light weight gears of thermoplastic materials.

Die casting:

The die casting method, the shape of gears depends on the Die shape. It is similar to Melton metal casting. Brass, aluminium, Zince and magnesium gear are made by this process.

Sintering:

It is used to manufacture of small and heavy duty gears for pumps and instrument. Iron and brass material are most common material of sintering method.

Investment casting:

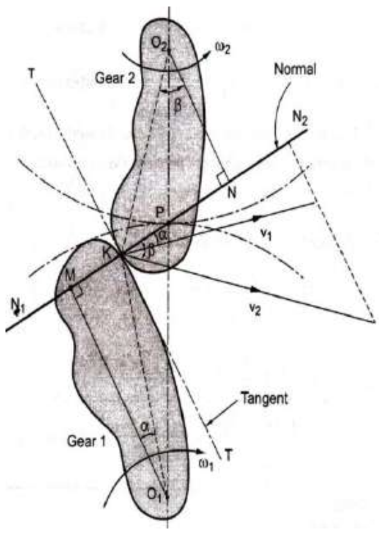
It can produce medium duty gears of iron and steel for rough application

Law of Gearing

Law of gearing: *The law of gearing states that for maintaining constant angular velocity ratio between two meshing gears, the common normal of the tooth profiles, at all contact points within mesh, must always pass through a fixed point on the lines of centres, called* ***pitch point.***

Proof: Consider portions of two gears 1 and 2 such that gear 1 rotates at a speed of ω_1 about its axis O_1 while gear 2 rotates with speed ω_2 about the axis O_2, as shown in Fig.

Let at any instant, K may be their common point of contact. Draw a common tangent TT at the point of contact K. Also draw a normal N_1N_2 to the tangent at K which intersects the line connecting the two axes of rotation at P, as shown. Join O_1 with K and from O_1, draw a perpendicular O_1M. Similarly, for gear 2, join O_2 with K. Then draw a perpendicular O_2N.



Let v_1 and v_2 be the velocities at the point of contact K of the gears 1 and 2 respectively.

Then $\quad v_1 = \omega_1 \times O_1K$

and $\quad v_2 = \omega_2 \times O_2K$

If the two gears 1 and 2 are always to remain in contact, the components of v_1 and v_2 along the line of transmission of motion (*i.e.*, along the common normal N_1N_2) must be equal at any instant.

$$\left.\begin{array}{c}\text{Component of } v_1 \text{ along}\\ \text{common normal } N_1N_2\end{array}\right\} = \left\{\begin{array}{c}\text{Component of } v_2 \text{ along}\\ \text{common normal } N_1N_2\end{array}\right.$$

$$v_1 \cos\alpha = v_2 \cos\beta$$

$$(\omega_1 \times O_1K)\cos\alpha = (\omega_2 \times O_2K)\cos\beta \qquad \ldots \text{(i)}$$

But from ΔO_1MK and ΔO_2NP, we get

$$\cos\alpha = \frac{O_1M}{O_1K} \quad \text{and} \quad \cos\beta = \frac{O_2N}{O_2K}$$

Substituting the values of $\cos\alpha$ and $\cos\beta$ in the equation (i), we have

$$(\omega_1 \times O_1K) \times \frac{O_1M}{O_1K} = (\omega_2 \times O_2K) \times \frac{O_2N}{O_2K}$$

or

$$\omega_1 \times O_1M = \omega_2 \times O_2N$$

or

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} \quad \dots \text{(ii)}$$

From similar ΔO_1MP and ΔO_2NP, we get

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \quad \dots (iii)$$

From equations (*ii*) and (*iii*), we can write

$$\frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} \quad \dots \text{(iv)}$$

Thus the ratio of angular velocities (ω_1/ω_2) varies inversely with the ratio of distances of point P from centres O_1 and O_2. It should be noted the ratio (ω_1/ω_2) will remain constant as long as the position of point P is fixed along the line O_1O_2.

Conclusion: Therefore, *in order to have a constant angular velocity ratio for all positions of the gears, the common normal at the point of contact between a pair of teeth must always pass through the pitch point.*

In other words, any tooth shape should fulfill the following conditions for the velocity ratio to remain constant:

(*i*) The path of gear contact is along a straight line known as line of action.

(*ii*) The line of action should pass through the pitch point.

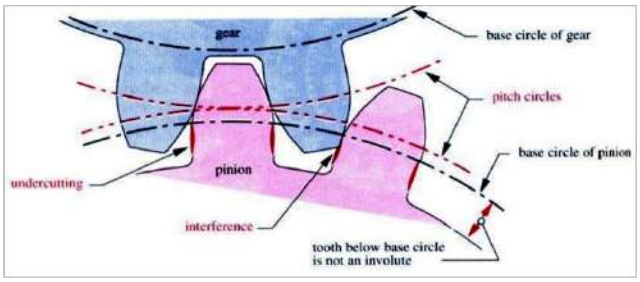
(*iii*) The pitch point must remain stationary on the line of centres.

Interference

- The **mating of two non involutes** tooth profiles is known as **interference**

Undercutting in gears

- When the tip of the tooth undercuts the root (flank) of the mating gear tooth, some portion of the flank will be removed. This process of removal of material due to interference phenomenon is called undercutting.



Methods to Avoid Interference

- By modifying addendum of gear teeth
- By increasing the pressure angle
- By modifying tooth profile or tooth shifting
- By increasing the centre distance
- By increasing no. of teeth on the mating pinion
- By Undercutting the radial flank of the pinion

Problem

1. **A pair of gears having 40 and 30 teeth respectively are 25° involutes form. The addendum length is 5mm and module pitch is 2.5mm. If the smaller wheel is driver and rotates at 1500 rpm, find the velocity of sliding (i) At the point of engagement (ii) At the point of disengagement (iii) At the pitch point.**

☹ ***Given data:*** $T_G = 40$; $T_P = 30$; $\phi = 25°$; Addendum, $a_P = a_W = 5$ mm; $m = 2.5$ mm; $N_P = 1500$ rpm.

☺ ***Solution:*** Angular velocity of pinion, $\omega_P = \frac{2\pi N_P}{60} = \frac{2\pi \times 1500}{60} = 157.08$ rad/s

$$\text{Gear ratio, } \frac{\omega_P}{\omega_G} = \frac{T_G}{T_P} \text{ or } \frac{157.08}{\omega_G} = \frac{40}{30} \text{ or } \omega_G = 117.81 \text{ rad/s}$$

Pitch circle radii of pinion and gear are given by

$$r = \frac{m\,T_P}{2} = \frac{2.5 \times 30}{2} = 37.5 \text{ mm}$$

and $$R = \frac{m\,T_G}{2} = \frac{2.5 \times 40}{2} = 50 \text{ mm}$$

Addendum circle radii of pinion and gear are given by

$$r_A = r + a_P = 37.5 + 5 = 42.5 \text{ mm}$$

and $$R_A = R + a_W = 50 + 5 = 55 \text{ mm}$$

Length of path of approach, $KP = \sqrt{R_A{}^2 - R^2 \cos^2\phi} - R\sin\phi$

or $$KP = \sqrt{(55)^2 - (50)^2 \cos^2 25°} - 50 \sin 25° = 10.04 \text{ mm}$$

Length of path of recess, $PL = \sqrt{r_A{}^2 - r^2 \cos^2\phi} - r\sin\phi$

or $$PL = \sqrt{(42.5)^2 - (37.5)^2 \cos^2 25°} - 37.5 \sin 25° = 9.67 \text{ mm}$$

(i) Velocity of sliding at the point of engagement (v_{sK}):

We know that velocity of sliding at the point of engagement K,

$$v_{sK} = (\omega_1 + \omega_2) \times \text{Length of path of approach} = (\omega_1 + \omega_2)\mathbf{KP}$$
$$= (157.08 + 117.81) \times 10.04 = 2759.89 \textbf{ mm/s or 2.75 m/s}$$

(ii) Velocity of sliding at the point of disengagement (v_{sL}):

We know that the velocity of sliding at the point of disengagement, L

$$v_{sL} = (\omega_1 + \omega_2) \times \text{Length of path of recess} = (\omega_1 + \omega_2)\,\mathbf{PL}$$
$$= (157.08 + 117.81) \times 9.67 = \mathbf{2658\ mm/s\ or\ 2.66\ m/s}$$

(iii) Velocity of sliding at the pitch point (v_{sP}):

The velocity of sliding at the pitch point is zero

GEAR TRAIN

It's a combination of wheels by means of which motion is transmitted from one shaft to another shaft

$$\boxed{\text{Speed ratio} = \frac{\text{Speed of the driving gear}}{\text{Speed of the driven gear}}}$$

$$\boxed{\text{Train value} = \frac{1}{\text{Speed ratio}} = \frac{\text{Speed of the driven gear}}{\text{Speed of the driving gear}}}$$

Problems

1. **Two parallel shafts are connected by a simple gear train. The speed of the driving gear and the driven gear is 360 rpm and 120 rpm respectively. The number of teeth on the driving gear is 30. Determine i) Speed ratio ii) The Train Value iii) Number of teeth on the driven or Follower**

Given Data : Simple gear train :

$$N_1 = 360 \text{ r.p.m.}$$
$$N_2 = 120 \text{ r.p.m.}$$
$$T_1 = 30$$

☺ ***Solution :***

(i) Speed Ratio :

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{360}{120} = \mathbf{3}\ \ \textbf{Ans.}$$

(ii) Train value :

$$\text{Train value} = \frac{1}{\text{Speed ratio}} = \frac{N_2}{N_1} = \frac{1}{3} = \mathbf{0.3333}\ \ \textbf{Ans.}$$

(iii) Number of teeth on driven gear :

We know that the speed ratio,

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

$$\frac{360}{120} = \frac{T_2}{30} \quad \text{or} \quad T_2 = \mathbf{90}\ \ \textbf{Ans.}$$

A compound gear train consists of six gears. The number of teeth on the gears are as follows :

Gear	*A*	*B*	*C*	*D*	*E*	*F*
No. of teeth	*60*	*40*	*50*	*25*	*30*	*24*

The gears B and C are on one shaft while the gears D and F are on another shaft. The gear A drives gear B, gear C drives gear D and gear E drives gear F. If the speed of the gear A is 100 r.p.m., then determine the speed of the gear F.

Given Data : $T_A = 60$; $T_B = 40$;

$T_C = 50$; $T_D = 25$;

$T_E = 30$; $T_F = 24$;

$N_A = 100$ r.p.m.

☺ ***Solution :*** We know that for a compound gear train,

$$\frac{\text{Speed of the first driver}}{\text{Speed of the last driven}} = \frac{\text{Product of number of teeth on the drivens}}{\text{Product of number of teeth on the drivers}}$$

i.e.,

$$\frac{N_A}{N_F} = \frac{T_B \times T_D \times T_F}{T_A \times T_C \times T_E}$$

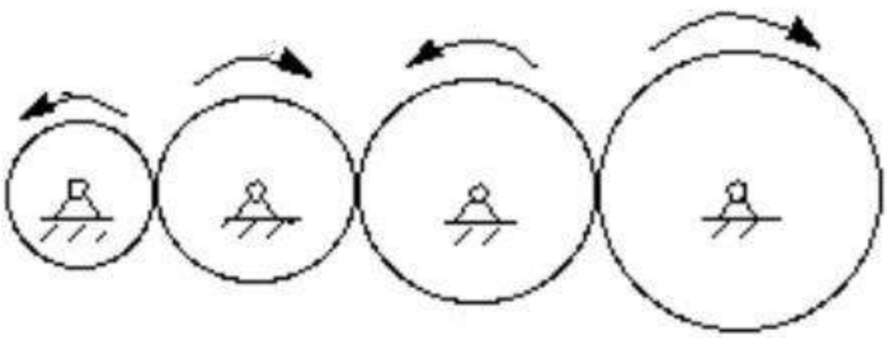
$$\frac{100}{N_F} = \frac{40 \times 25 \times 24}{60 \times 50 \times 30}$$

Speed of the gear F, N_F = **375 r.p.m. Ans.**

Simple gear train

Characteristics of simple gear train

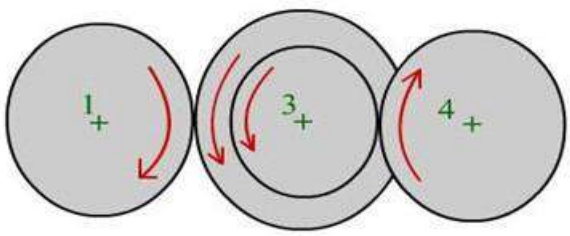
- Gears are in the form of series
- Axes of gears remains fixed
- Each gear is mounted on different shaft



Compound gear train

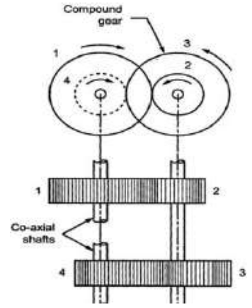
Characteristics of compound gear train

- All the characteristics are same as simple gear train except that more than two gears can also be mounted on a single shaft



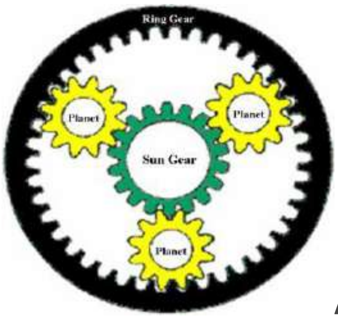
Reverted Gear Train

When the axes of the first gear (i.e., first driver) and the last gear (i.e., last driven or follower) are co-axial, then the gear train is known as reverted gear train.



EpiCyclic Gear Train

A gear train that consists of one or more outer gears rotating around a central gear. Epicylic gear trains are also known as planetary gear trains.

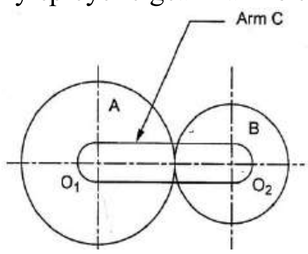


- ✓ Differential gears of automobile,
- ✓ Back gear of lathe,
- ✓ Pulley blocks,
- ✓ Hoists and
- ✓ Writs watches.

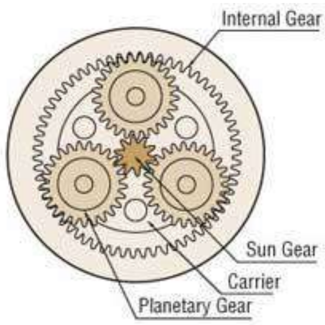
Applications

Types

a) **Simple Epyclic Gear Train**
When only one gear on each shaft in any epicyclic gear train is called simple epicyclic gear train



b) **Compound Epyclic Gear Train**
When there are more than one gear on a shaft in any epicyclic gear train is called as compound epicyclic gear train



Cross Section of a **PN** Gear

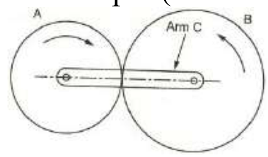
Methods to find Velocity Ratio

1. Tabulation Method
2. Algebraic or Formula Method

In an epicyclic gear train, an arm carries two gears A & B having 24 & 30 teeth respectively. The arm rotates at 100 rpm in the clockwise direction. Find the speed of the gear B on its own axis, when the gear A is fixed. If instead of being fixed, the wheel A rotates at 200 rpm in the counter clockwise direction, what will the speed of B?

Given Data:

$T_A = 24$: $T_B = 30$: $N_C = 100$ rpm (Clockwise)



S.No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm C fixed; Gear A rotates through −1 revolution *i.e.*, 1 revolution clockwise.	0	−1	$+\frac{T_A}{T_B}$
2.	Arm C fixed; Gear A rotates through $-x$ revolutions	0	$-x$	$+x \times \frac{T_A}{T_B}$
3	Add $-y$ revolutions to all elements	$-y$	$-y$	$-y$
4	Total motion	$-y$	$-x-y$	$x \times \frac{T_A}{T_B} - y$

Speed of gear B when gear A is fixed : Since the speed of arm is 100 r.p.m. clockwise, therefore from the fourth row of the table,

$$-y = -100 \quad \text{or} \quad y = 100 \text{ r.p.m.}$$

$\because$ gear A is fixed, therefore

$$-x - y = 0 \quad \text{or} \quad x = -y = -100 \text{ r.p.m.}$$

Hence, Speed of gear B $= x \frac{T_A}{T_B} - y$ ——— From Table

$$= -100 \times \frac{24}{30} - 100 = -180 \text{ r.p.m.}$$

i.e., Speed of gear B = **180 r.p.m. (clockwise) Ans.**

Speed of gear B when gear A rotates 200 r.p.m. counter clockwise :

Since the gear A rotates 200 r.p.m. counter clockwise, therefore from the fourth row of the table,

$$-x - y = +200 \quad \text{or} \quad x + y = -200$$

$$\therefore x = -200 - y$$

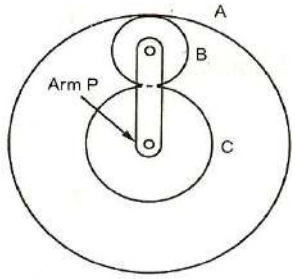
$$= -200 - 100 = -300$$

Hence speed of gear B, $N_B = x \frac{T_A}{T_B} - y$

$$= -300 \times \frac{24}{30} - 100 = -340$$

i.e., Speed of gear B = **340 r.p.m. (clockwise) Ans.**

2. **In an epicyclic gear train an annular wheel A having 54 teeth meshes with a planet wheel B which gears with a sun wheel C, the wheels A &C beings coaxial. The wheel B is carried on a pin fixed on end of the arm P which rotates about the axis of the wheels A & C. If the wheel A makes 20 rpm in a clockwise sense and the arm rotates at 100 rpm in the anticlockwise direction and the wheel C has 24 teeth, determine the speed and sense of rotation of wheel C?**



Given Data : $T_A = 54$; $N_A = 20$ r.p.m. (clockwise)

$N_{arm} = 100$ r.p.m. (anticlockwise) ; $T_C = 24$

☺ ***Solution :*** The given gear arrangement is shown in Fig. The motion of various elements is shown in Table

Table ***Motion of Elements***

Step No.	Conditions of motion	Revolutions of elements			
		Arm P	Gear C	Gear B	Gear A
1.	Arm P fixed; Gear C rotates through +1 revolution. (*i.e.*, 1 rev. anticlockwise)	0	+1	$\frac{-T_C}{T_B}$	$\frac{-T_C}{T_B} \times \frac{T_B}{T_A} = \frac{-T_C}{T_A}$
2.	Arm P fixed; Gear C rotates through $+x$ revolution.	0	$+x$	$-x\frac{T_C}{T_B}$	$-x\frac{T_C}{T_A}$
3.	Add $+y$ revolutions to all elements.	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\frac{T_C}{T_B}$	$y-x\frac{T_C}{T_A}$

The given conditions are

(i) Wheel A makes 20 r.p.m. in clockwise,

so, $N_A = -20$ r.p.m.

$$\therefore\ y - x\frac{T_C}{T_A} = -20 \qquad \text{[From Table]} \quad \dots \text{(i)}$$

(ii) Arm P makes 100 r.p.m. in anticlockwise,

so, $y = +100$ r.p.m. ... (ii)

From conditions (i) and (ii), we get

$$100 - x \times \frac{24}{54} = -20$$

or $x = 270$ r.p.m.

$\therefore$ Speed of gear wheel C, $N_C = x + y$ [From Table]

$N_C = 270 + 100 = +370$ r.p.m.

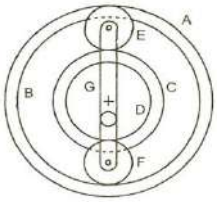
$\therefore$ N_C = **370 r.p.m. (anticlockwise sense) Ans.**

In an epicyclic gear train the internal wheels A & B are compound wheels C & D rotate independently about axis O. the wheel E & F rotate on pins fixed on the arm G. E gears with A & C. Wheel F gears with B & D. All the wheels have the same module and the number of teeth are T_C=28; T_D = 26; TE = T_F= 18.

(1) Sketch the arrangement and find the number of teeth on A & B.

(2) If the arm G makes 100 rpm clockwise and A is fixed, find the speed of B.

(3) If the arm G makes 100 rpm clockwise and wheel A make 10 rpm counter clockwise, find the speed of B.



3. If the arm G makes 100 r.p.m. clockwise and A is fixed, find the speed of B.

4. If the arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise; find the speed of wheel B.

Given Data : $T_C = 28$; $T_D = 26$; $T_E = T_F = 18$

☺ ***Solution :***

1. Sketch the arrangement : The gear arrangement is shown in Fig.

2. Number of teeth on wheels A and B : From the geometry of Fig. we can write

$$d_A = d_C + 2\,d_E$$

and

$$d_B = d_D + 2\,d_F$$

where d_A, d_B, d_C, d_D, d_E and d_F are the pitch circle diameter of wheels A, B, C, D, E and F respectively.

We know that for the same module, the number of teeth are proportional to the pitch circle diameters, so

$$T_A = T_C + 2\,T_E = 28 + 2 \times 18 = \mathbf{64} \quad \textbf{Ans.}$$

and

$$T_B = T_D + 2\,T_F = 26 + 2 \times 18 = \mathbf{62} \quad \textbf{Ans.}$$

3. Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A is fixed :

The motion of various elements is shown in Table

Motion of Elements

Step No.	Conditions of motion	Revolutions of elements					
		Arm G	Wheel A	Wheel E	Compound wheel C-D	Wheel F	Wheel B
1.	Arm fixed; Wheel A rotates through +1 revolution (*i.e.*, 1 rev. anticlockwise)	0	+1	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E} \times \frac{T_E}{T_C}$ $= -\frac{T_A}{T_C}$	$+\frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$+\frac{T_A}{T_C} \times \frac{T_D}{T_F} \times \frac{T_F}{T_B}$ $= +\frac{T_A}{T_C} \times \frac{T_D}{T_B}$
2.	Arm fixed; Wheel A rotates through $+x$ revolutions.	0	$+x$	$+x \times \frac{T_A}{T_E}$	$-x \times \frac{T_A}{T_C}$	$+x + \frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$+x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$
3.	Add $+y$ revolutions to all elements.	$+y$	$+y$	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x + y$	$y+x\times\frac{T_A}{T_E}$	$y - x \times \frac{T_A}{T_C}$	$y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$

Given conditions are

(i) G makes 100 r.p.m. clockwise, so $y = -100$

(ii) Wheel A is fixed, so $x + y = 0$ or $x = -y = 100$

$$\therefore \text{Speed of wheel B} = y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B} \qquad \text{[From Table}$$

$$= -100 + 100 \times \frac{64}{28} \times \frac{26}{62} = -4.2 \text{ r.p.m.}$$

$\therefore N_B =$ **42 r.p.m. (clockwise) Ans.**

4. Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter clockwise :

The given conditions are

(i) Arm G makes 100 r.p.m. clockwise, so $y = -100$

(ii) Wheel A makes 10 r.p.m. counterclockwise, so $x + y = 10$

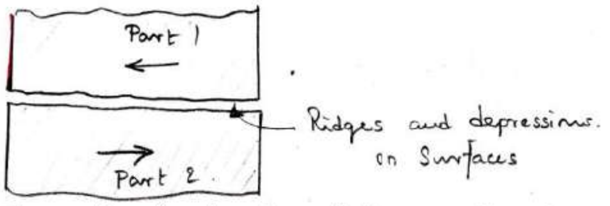
or $x = 10 - y = 10 + 100 = 110$

$$\therefore \text{Speed of wheel B} = y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B} \qquad \text{[From Table}$$

$$= -100 + 100 \times \frac{64}{28} \times \frac{26}{62} = +5.4 \text{ r.p.m.}$$

$\therefore N_B =$ **5.4 r.p.m. (counter clockwise) Ans.**

Unit 5- FRICTION IN MACHINE ELEMENTS



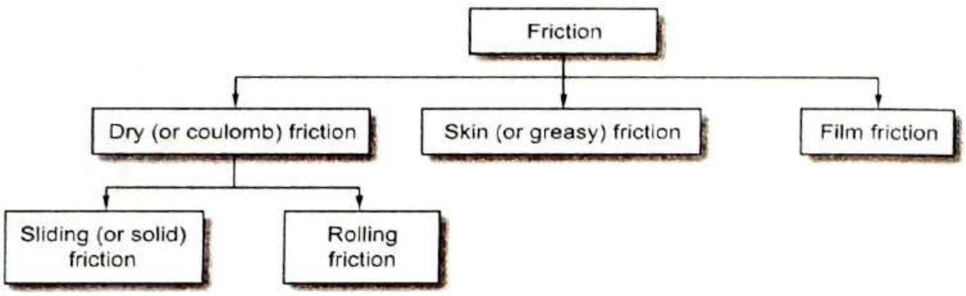
Smoothness of surface is only a relative term. Even the most accurately planned surface will not be free from ridges and depressions which may not be visible to the naked eye, but they become apparent on microscopical examination. When one surface has contact with another surface, these ridges and depressions interlock and thus the relative motion of one surface over the other is resisted. This resistive force is known as the *force of friction* or ***frictional force.*** *Force of friction is always acting in the direction opposite to the direction of motion.*

Types of Surfaces :

(i) Dry surfaces,

(ii) Greasy or partially lubricated surfaces, and

(iii) Completely lubricated or film lubricated surfaces.



Dry Friction

The friction that exists between two unlubricated surfaces is known as solid friction or dry friction.

Laws of Dry Friction

1. The frictional force (force of friction) is directly proportional to the normal reaction between the surfaces.
2. The frictional force opposes the motion or its tendency to the motion.
3. The frictional force depends upon the nature of the surfaces in contact.
4. The frictional force is independent of the area and the shape of the contacting surfaces.
5. The frictional force is independent of the velocity of sliding of one body relative to the other body.

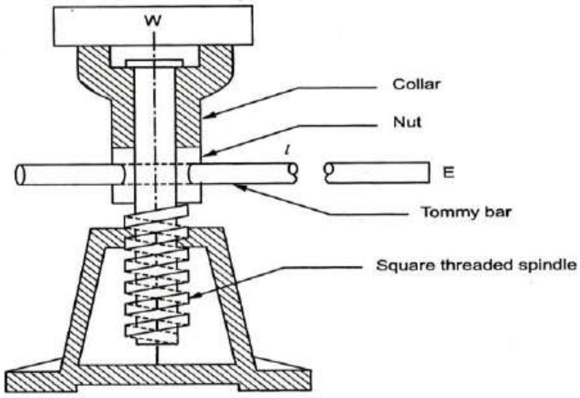
Limiting Friction

The ultimate (maximum) value of frictional force, which comes into play, when a body just tends to move, is known as ***limiting force of friction*** or ***limiting friction*** or ***friction of impending sliding.*** In other words, the maximum value of frictional force acting on a body, when the body is on the point of motion, is called limiting force of friction.

Coefficient of friction, $\boxed{\mu = \frac{\text{Limiting force of friction}}{\text{Normal reaction}} = \frac{F}{R_N}}$

Forms of Screw and Nut

1. In screw pairs, the nut forms a fixed part and the screw on rotating advances or recedes in the direction of its axis. ***Example :*** Screw jack



2. In other case, the screw forms a turning part and nut a sliding part. As the screw rotates, the nut moves along the axis of the screw. ***Example :*** Lead screw of lathe.

Types of Threads

1. According to the shape of the threads :

(a) Square threads,

(b) V-threads.

2. According to the surface of the threads :

(a) External threads : When threads are cut on the outer surface of a solid rod, then they are known as external threads.

(b) Internal threads : When threads are cut on the internal surface of the hollow rod, then they are known as internal threads.

3. According to the number of threads :

(a) Single threaded screw : If only one thread is cut in one lead distance of screw, it is known as single threaded screw.

(b) Multi threaded screw : If more than one thread is cut in one lead distance of a screw, it is known as multi-threaded screw.

4. According to the direction of movement of threads :

(a) Right handed screws

(b) Left handed screws

As they move from or towards the observer looking at one end when they are rotated in clockwise direction to a fixed nut, we may get right handed or left handed screws.

Terminology Used in Lead Screw

1. ***Helix :*** It is the curve traced by a particle while moving along a screw thread.
2. ***Pitch :*** It is the axial distance from a point on a screw threads to a corresponding point on the next thread.
3. ***Lead :*** It is the distance, a screw thread advances axially in one turn.
4. ***Depth of thread (crest or root of a thread) :*** It is the distance between the top and bottom surfaces of a thread.

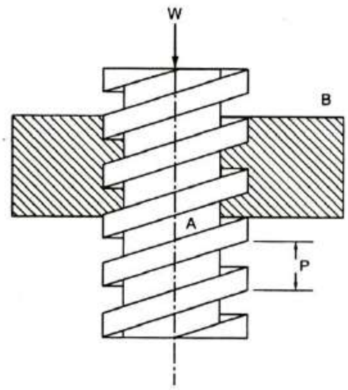
5. ***Single threaded screw :*** If the lead of screw is equal to its pitch, it is known as single threaded screw.

6. ***Multi-threaded screw :*** If more than one thread is cut in one lead screw of a screw, it is known as multi-threaded screw.

   $\therefore$ Lead = Pitch $\times$ Number of threads

7. ***Slope of thread :*** It is the inclination of the thread with the horizontal.

   $$\therefore \tan\alpha = \frac{\text{Lead of screw}}{\text{Circumference of screw}}$$



Problems

1. **A bolt with a square threaded screw has mean diameter of 25 mm and a pitch of 3 mm. It carries axial thrust of 10 KN on the bolt head of 25 mm mean radius. If coefficient of friction is 0.12, find the force required at the end of a spanner 450 mm long in tightening up the bolt.**

Given Data : Square thread ;

Mean dia. $d = 25 \text{ mm} = 25 \times 10^{-3}$ m ;

$p = 3 \text{ mm} = 3 \times 10^{-3}$ m ;

$W = 10 \text{ KN} = 10 \times 10^{3}$ N ;

Bolt head radius $R = 25$ mm ;

$\mu = \tan\phi = 0.12$;

$l = 450 \text{ mm} = 0.45$ m

To find : Force required at the end of a spanner

☺ ***Solution :***

$$\tan\alpha = \frac{p}{\pi d} = \frac{3 \times 10^{-3}}{\pi \times 25 \times 10^{-3}} = 0.00382$$

Force at the circumference of screw is given by

$$P = W\tan(\alpha+\phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \cdot \tan\phi}\right]$$

$$= 10 \times 10^{3}\left[\frac{0.00382 + 0.12}{1 - 0.00382 \times 0.12}\right]$$

$$P = 1589.26 \text{ N}$$

Total torque required, $T = P \times \frac{d}{2} + \mu W R$

$$= 1589.26 \times \frac{25 \times 10^{-3}}{2} + 0.12 \times 10 \times 10^{3} \times 25 \times 10^{-3}$$

$$T = 49.866 \text{ N-m}$$

Force at the end of spanner,

$$F = \frac{\text{Torque}}{\text{Length of lever}}$$

$$= \frac{49.866}{0.45} = \mathbf{110.81\ N\ Ans.}$$

2. The follower data relates to screw jack:

Pitch of the threaded screw = 8mm

Diameter of the threaded screw = 40 mm

Coefficient of friction between screw and nut = 0.1

Load = 20 kW

Assuming that load rotates with the screw, determine

1. The ratio of torques required to raise and lower the load

2. The efficiency of the machine

Given Data : Square thread ;

Mean dia. $d = 25 \text{ mm} = 25 \times 10^{-3} \text{ m}$;

$p = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$;

$W = 10 \text{ KN} = 10 \times 10^{3} \text{ N}$;

Bolt head radius $R = 25 \text{ mm}$;

$\mu = \tan\phi = 0.12$;

$l = 450 \text{ mm} = 0.45 \text{ m}$

To find : Force required at the end of a spanner

☺ ***Solution :*** $\tan\alpha = \frac{p}{\pi d} = \frac{3 \times 10^{-3}}{\pi \times 25 \times 10^{-3}} = 0.00382$

☺ ***Solution :*** Helix angle is given by

$$\tan\alpha = \frac{p}{\pi d} = \frac{8 \times 10^{-3}}{\pi \times 40 \times 10^{-3}} = 0.0637$$

$$\alpha = 3.64°$$

Friction angle ϕ is given by

$$\mu = \tan\phi = 0.1$$

$$\phi = 5.71°$$

(i) Ratio of torques required to raise and lower the load :

$$\text{Torque required to raise the load } T_1 = W \tan(\alpha + \phi) \cdot \frac{d}{2}$$

$$= 20 \times 10^{-3} \tan(3.64° + 5.71°) \cdot \frac{40 \times 10^{-3}}{2}$$

$$T_1 = 65.861 \text{ N-m}$$

$$\text{Torque required to lower the load, } T_2 = W \tan(\phi - \alpha)\ \frac{d}{2}$$

$$= 20 \times 10^{3} \tan(5.71° - 3.64°) \times \frac{40 \times 10^{-3}}{2}$$

$$T_2 = 14.457 \text{ N-m}$$

$$\text{Ratio of torques, } \frac{T_1}{T_2} = \frac{65.861}{14.457} = \mathbf{4.556}$$ **Ans.**

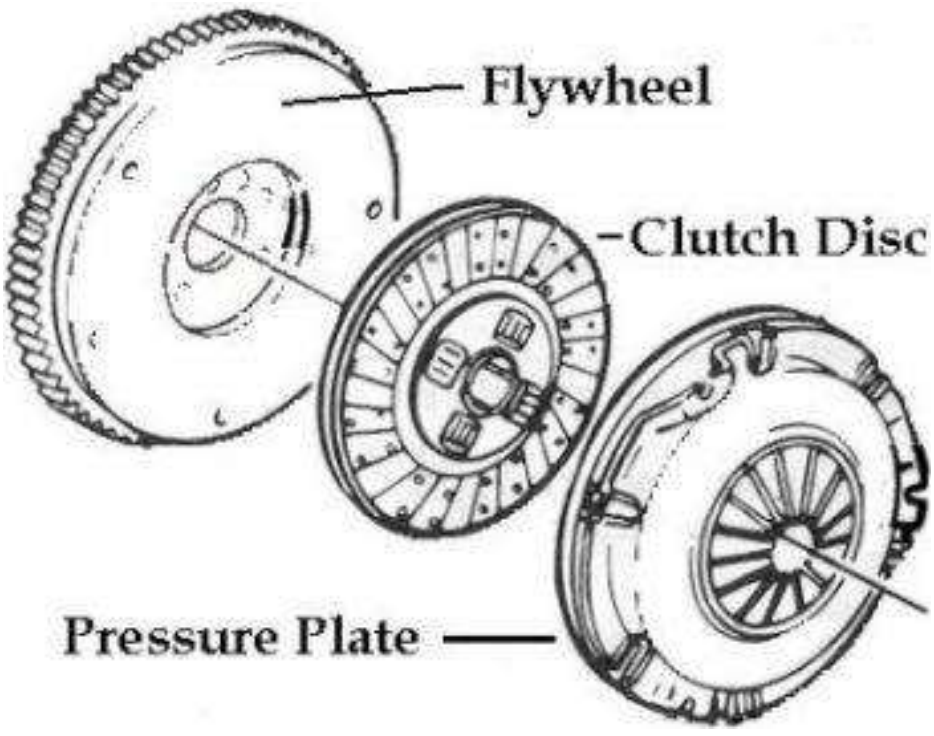
(ii) Efficiency of the machine :

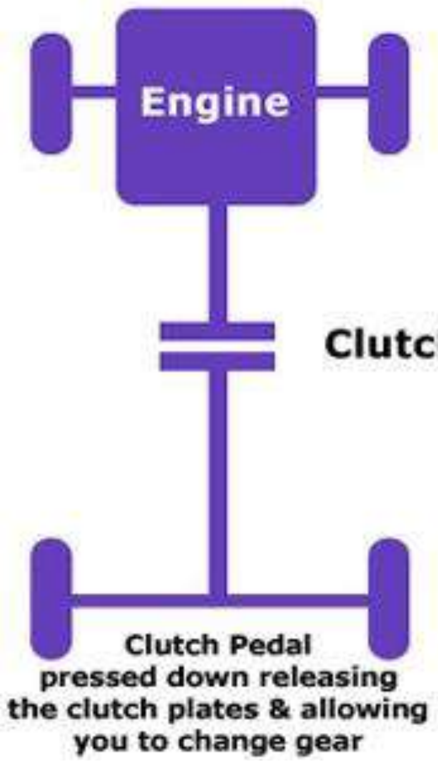
$$\eta_{screwjack} = \frac{\tan \alpha}{\tan(\alpha + \phi)} = \frac{\tan 3.64°}{\tan(3.64° + 5.71°)}$$

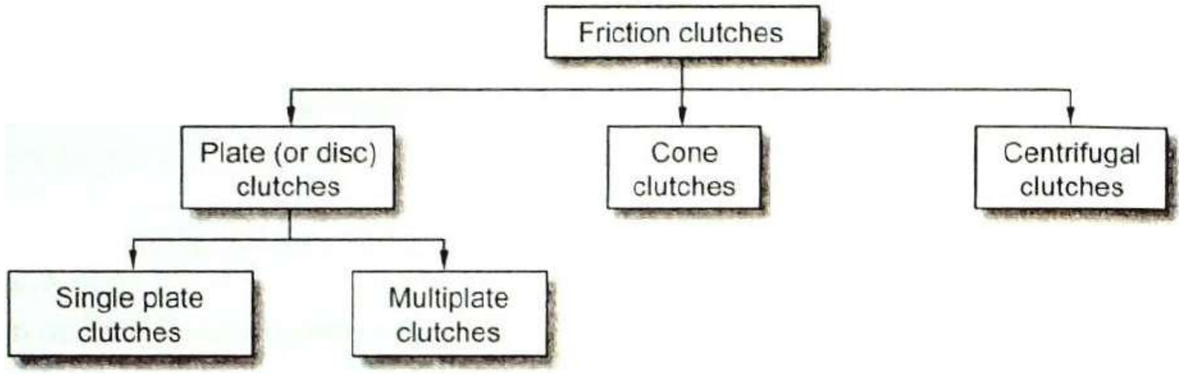
$$\eta = \mathbf{0.386 \text{ or } 38.6\%}$$ **Ans.**

CLUTCH

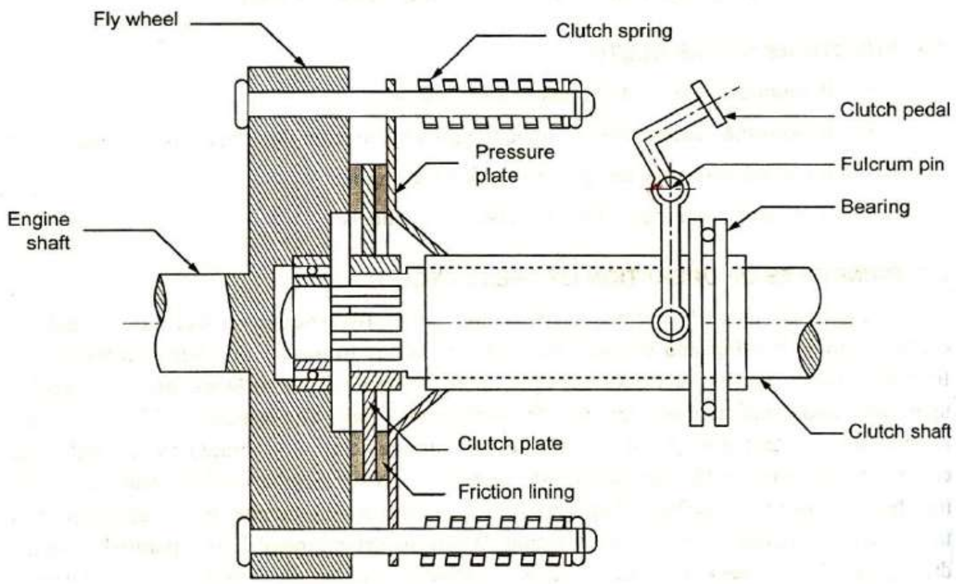
- A clutch is a mechanical device that engages and disengages the power transmission, especially from driving shaft to driven shaft.





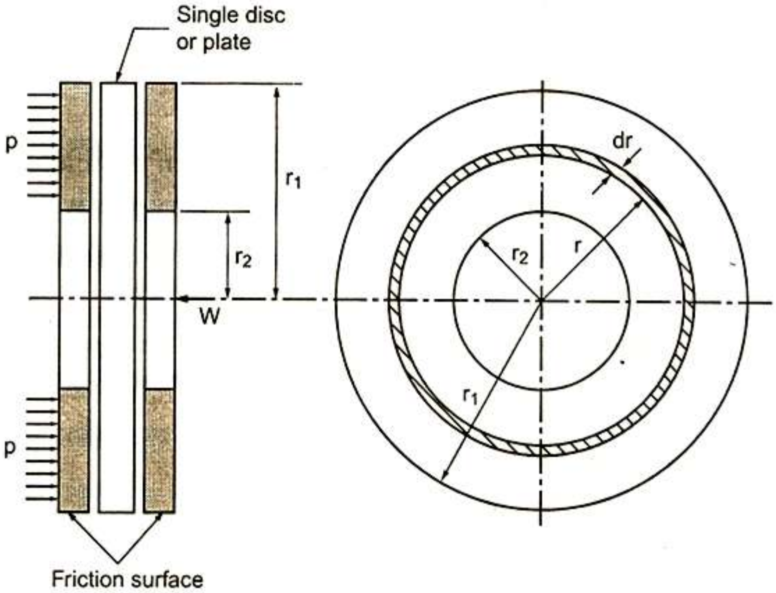


Single Plate Clutch



Torque transmitted by the Single Plate Clutch

Consider two friction surfaces are held together by an axial thrust W, as shown in Fig.



Let T = Torque transmitted by the clutch

p = Intensity of axial pressure acting on contact surfaces

r_1 = External radius of friction surface

r_2 = Internal radius of friction surface

μ = Coefficient of friction

Torque transmitted on Multiplate Clutch

Let n_1 = Number of discs on the driving shaft and

n_2 = Number of discs on the driven shaft.

$\therefore$ Number of pair of contact surfaces,

$$n = n_1 + n_2 - 1$$

Then, total frictional torque acting on the clutch is given by

$$\boxed{T = n \cdot \mu \cdot W \cdot R}$$

where R = Mean radius of the friction surfaces .

$$R = \frac{2}{3}\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right] \quad \text{[For uniform pressure]}$$

$$R = \frac{r_1 + r_2}{2} \quad \text{[For uniform wear]}$$

1. **An automotive single plate clutch consists of two pairs of contacting surfaces. The inner and outer radii of friction plate are 120 mm and 250 mm respectively. The coefficient of friction is 0.25 and the total axial force is 15kN.Calaulate the power transmitting capacity of the clutch plate at 500 r.p.m. using**

   (i) Uniform wear theory and (ii) Uniform pressure theory.

Given Data : $n = 2$; $r_1 = 250 \text{ mm} = 0.25 \text{ m}$;

$r_2 = 120 \text{ mm} = 0.12 \text{ m}$; $W = 15 \text{ kN} = 15 \times 10^3 \text{ N}$;

$N = 500 \text{ r.p.m.}$; $\mu = 0.25$

☺ ***Solution : (i) Using uniform wear theory :*** Torque transmitted on clutch is given by

$$T = n \cdot \mu \cdot W \frac{(r_1 + r_2)}{2}$$

$$= 2 \times 0.25 \times 15 \times 10^3 \frac{(0.25 + 0.12)}{2}$$

$$= 1387.5 \text{ N-m}$$

$$\therefore \text{Power transmitted} = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 500 \times 1387.5}{60} = 72649.33 \text{ W}$$

or $P = $ **72.65 kW Ans.** ☜

(ii) Using uniform pressure theory : Torque transmitted on clutch is given by

$$T = n \mu W \frac{2}{3} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right]$$

$$= 2 \times 0.25 \times 15 \times 10^3 \times \frac{2}{3} \times \left[\frac{(0.25)^3 - (0.12)^3}{(0.25)^2 - (0.12)^2} \right]$$

$$= 1444.6 \text{ N-m}$$

$$\therefore \text{Power transmitted} = \frac{2\pi N T}{60}$$

$$= \frac{2\pi \times 500 \times 1444.6}{60} = 75639 \text{ W}$$

$P = $ **75.64 kW Ans.** ☜

2. A single plate friction clutch, with both sides of the plate being effective, is used to transmit power at 1440 r.p.m. It has outer and inner radii 80 mm and 60 mm respectively. The maximum intensity of pressure is limited to 10 x 10^4 N/m^2. If the coefficient of friction is 0.3, determine (i)Total pressure exerted on the plate and (ii) Power transmitted.

Given Data : $n = 2$; $N = 1440 \text{ r.p.m.}$; $r_1 = 80 \text{ mm} = 80 \times 10^{-3} \text{ m}$;

$r_2 = 60 \text{ mm} = 60 \times 10^{-3} \text{ m}$; $p_{max} = 10 \times 10^4 \text{ N/m}^2$;

$\mu = 0.3$

☺ ***Solution : (i) Total pressure exerted on the plate :*** Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform wear

$$p_{max} \cdot r_2 = C \quad \text{or} \quad C = 10 \times 10^4 \times 60 \times 10^{-3} = 6000 \text{ N/m}$$

We know that the axial thrust,

$$W = 2\pi C (r_1 - r_2)$$

$$= 2\pi \times 6000 (80 \times 10^{-3} - 60 \times 10^{-3})$$

$$= 754 \text{ N}$$

$\therefore$ Axial thrust or total pressure exerted on the plate = **754 N Ans.** ☜

(ii) Power transmitted :

$$\text{Torque transmitted, } T = n \cdot \mu \cdot W \cdot \frac{(r_1 + r_2)}{2}$$

$$= 2 \times 0.3 \times 754 \frac{(80 \times 10^{-3} + 60 \times 10^{-3})}{2} = 31.67 \text{ N-m}$$

$$\text{Power transmitted, } P = \frac{2\pi N T}{60} = \frac{2\pi \times 1400 \times 31.67}{60} = 4643 \text{ W}$$

or $$P = \mathbf{4.643\ kW}\quad \textbf{Ans.}$$

3. A multiple clutch consisting of 6 plates, each plate of external diameter 150 mm and internal diameter 100 mm is to transmit 7.5 kW at 900 r.p.m. Assuming coefficient of friction is 0.1,determine the pressure on each effective pair of surfaces in contact.

Given Data : Number of plate, $n_p = 6$;

$$d_1 = 150 \text{ mm or } r_1 = 75 \text{ mm} = 75 \times 10^{-3} \text{ m}$$

$$d_2 = 100 \text{ mm or } r_2 = 50 \text{ mm} = 50 \times 10^{-3} \text{ m}$$

$$P = 7.5 \text{ kW} = 7.5 \times 10^3 \text{ W};$$

$$N = 900 \text{ r.p.m.};$$

$$\mu = 0.1$$

☺ ***Solution :*** As given, number of plates, $n_p = 6$,

$\therefore$ Number of pair of surfaces in contact, $n = n_p - 1 = 6 - 1 = 5$

$$\text{Power transmitted, } P = \frac{2\pi N T}{60}$$

$$7.5 \times 10^3 = \frac{2\pi \times 900 \times T}{60}$$

Total friction torque to be transmitted, $T = 79.57$ N-m

Since no assumption is given, we assume uniform wear.

$$T = \mu \cdot W \times \frac{(r_1 + r_2)}{2}$$

$$79.57 = 0.1 \times W \times \frac{(75 \times 10^{-3} + 50 \times 10^{-3})}{2}$$

$$W = 12731 \text{ N}$$

Total axial force or pressure on contact surfaces $= W = 12731$ N

But $n = 5$, so pressure on each effective surface $= \frac{12731}{5} = \mathbf{2546\ N}$ **Ans.**

4. **A multiple clutch has three discs on the driving shafts and two on the driven shafts. The outside diameter of the contact surfaces is 240 mm and inside diameter is 120 mm. Assume uniform wear coefficient of friction as 0.3, find the maximum axial intensity of pressure between the discs for transmitting 25 kW at 1575 r.p.m.**

Given Data : $n_1 = 3$; $n_2 = 2$;

$d_1 = 240$ mm or $r_1 = 120$ mm $= 0.12$ m ;

$d_2 = 120$ mm or $r_2 = 60$ mm $= 60 \times 10^{-3}$ m ;

$\mu = 0.3$; $P = 25$ kW $= 25 \times 10^3$ W ; N = 1775 r.p.m.

☺ ***Solution :*** Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1 = 3 + 2 - 1 = 4$$

$$\text{Power transmitted, } P = \frac{2\pi NT}{60}$$

$$25 \times 10^3 = \frac{2\pi\,(1575)\,T}{60}$$

$$T = 151.6 \text{ N-m}$$

Torque transmitted, for uniform wear is given by

$$T = n \cdot \mu \cdot W \left(\frac{r_1 + r_2}{2}\right)$$

$$151.6 = 4 \times 0.1 \times W \times \left(\frac{0.12 + 0.06}{2}\right)$$

$$W = 1404 \text{ N}$$

But axial force exerted is given by,

$$W = 2\pi C\,(r_1 - r_2)$$

$$W = 2\pi \times p_{max} \times r_2\,(r_1 - r_2) \qquad [\because C = p_{max} \times r_2]$$

$$1404 = 2\pi \times p_{max} \times 0.06\,(0.12 - 0.16)$$

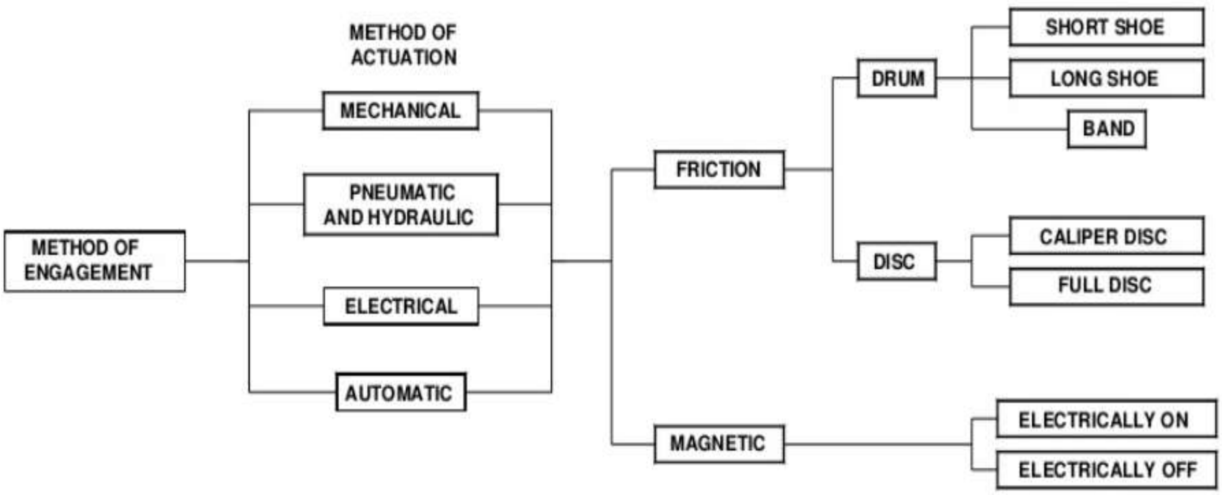
$$p_{max} = 62.07 \times 10^3 \text{ N/m}^2$$

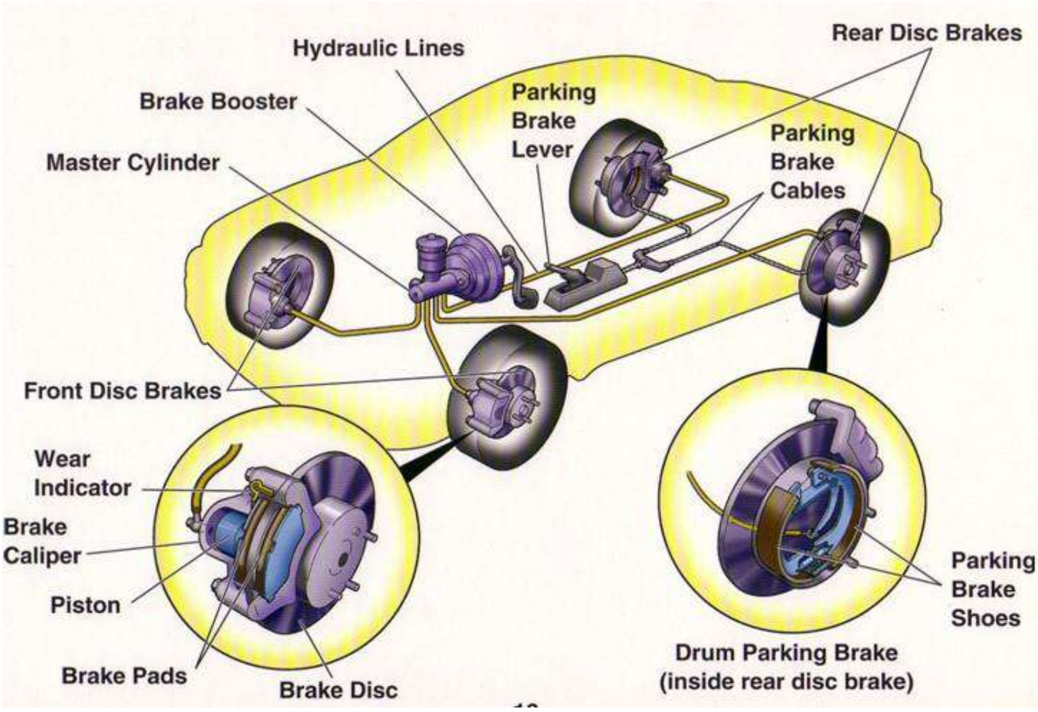
$$= \mathbf{62\ kN/m^2} \quad \textbf{Ans.}$$

BRAKES

- It's a mechanical device by means of which motion of a body is retarded for slowing down (or) to bring it to rest by applying artificial frictional resistance

Types of Brakes





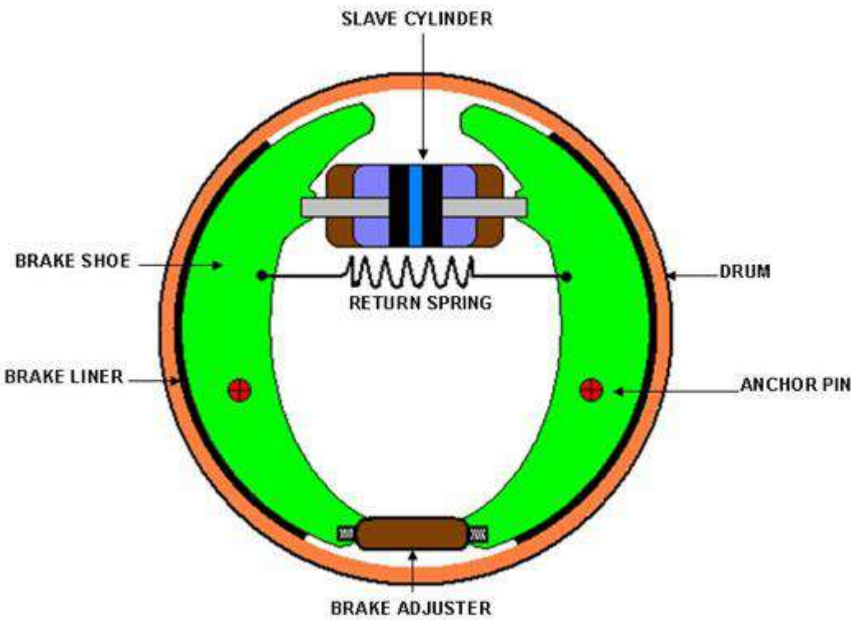
Advantages of Hydraulic Brakes

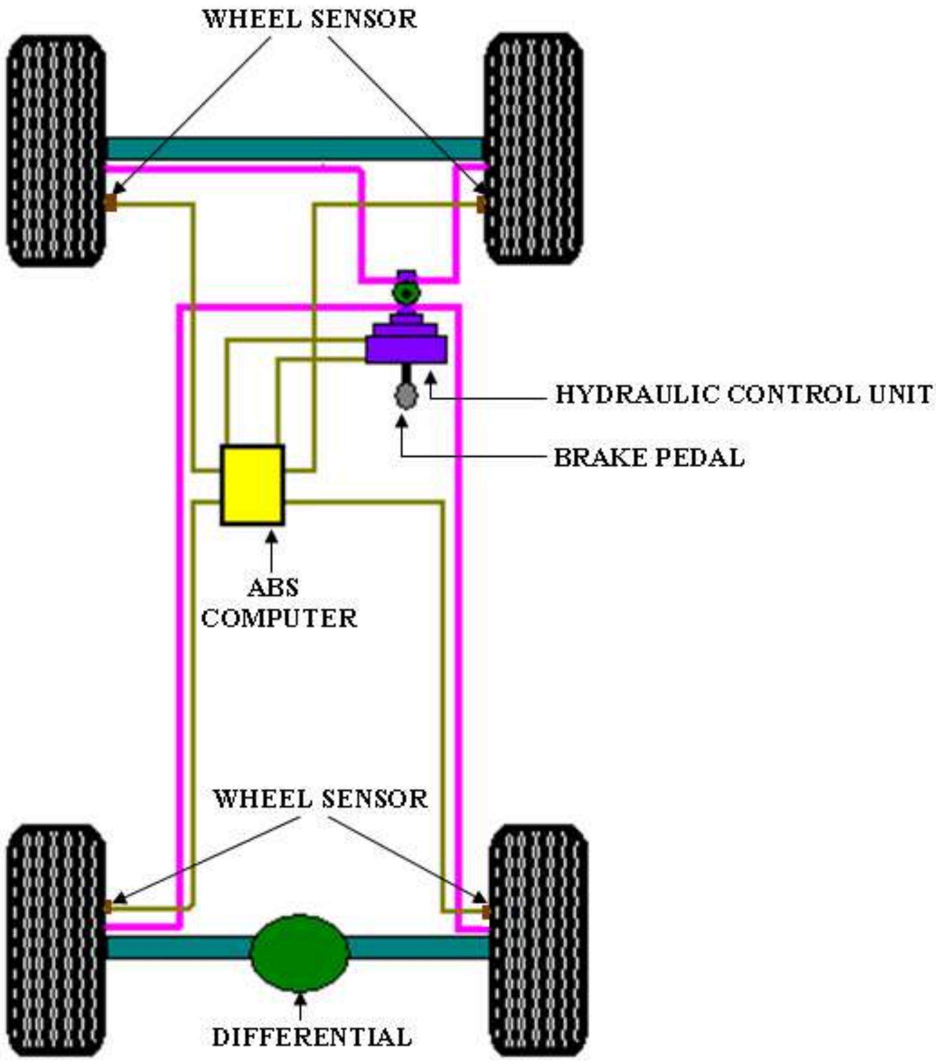
a) Equal braking action on all wheels.
b) Increased braking force.
c) Simple in construction.
d) Low wear rate of brake linings.
e) Flexibility of brake linings.
f) Increased mechanical advantage.

Disadvantages of Hydraulic Brakes

a) Whole braking system fails due to leakage of fluid from brake linings.
b) Presence of air inside the tubings ruins the whole system.

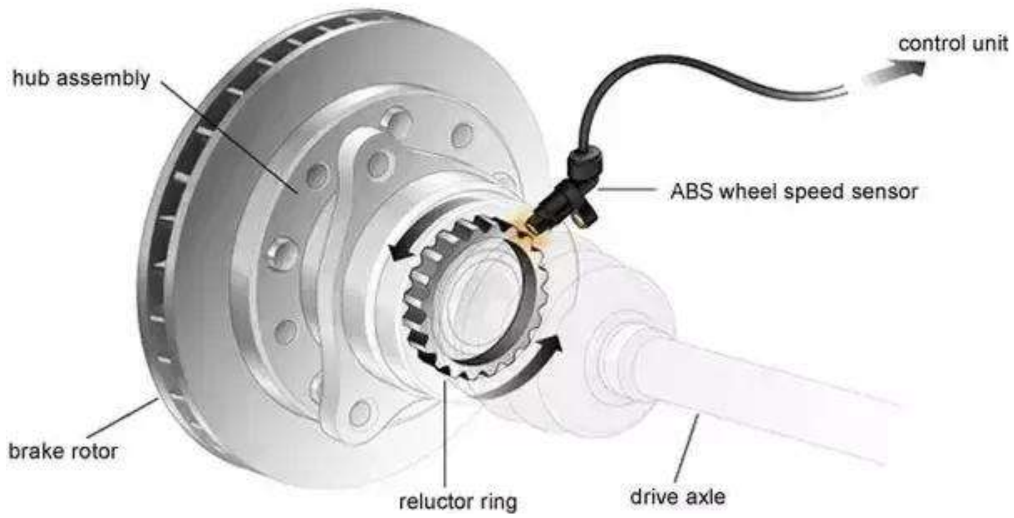
Drum Brake **Disc Brake**





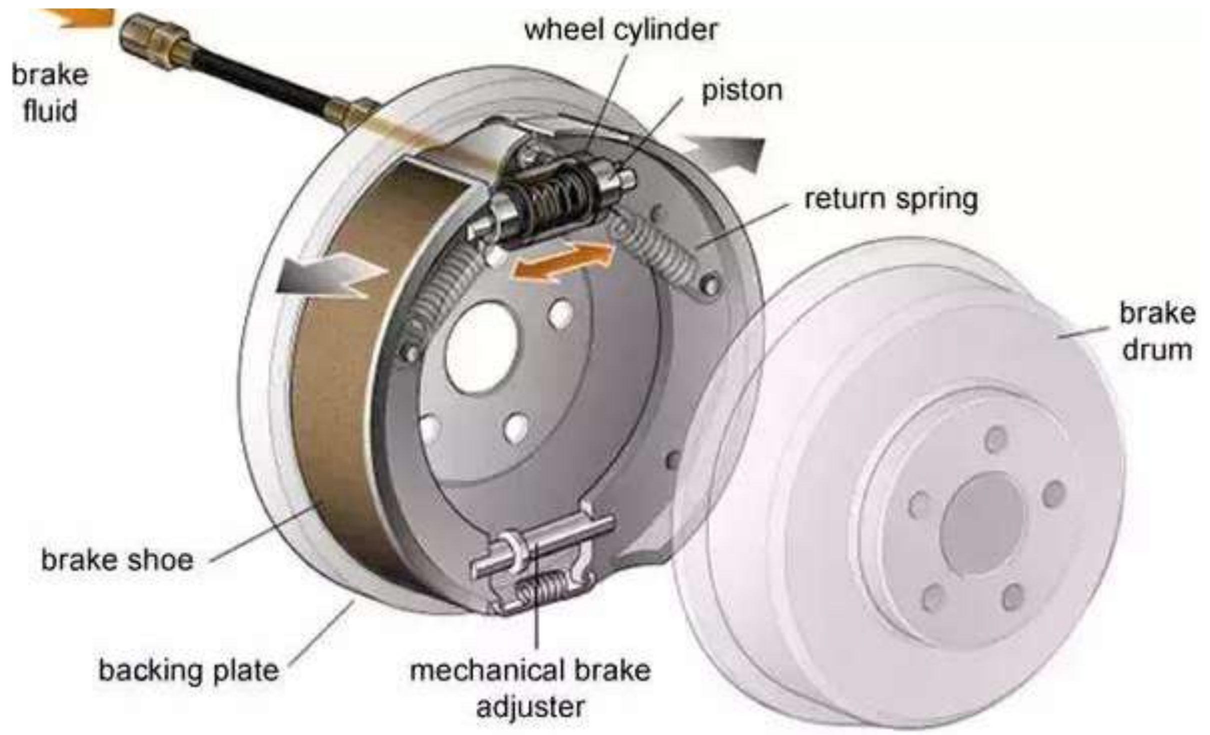
Disc Brakes

Disc brakes consist of a disc brake rotor - which is attached to the wheel - and a caliper, which holds the disc brake pads. Hydraulic pressure from the master cylinder causes the caliper piston to clamp the disc brake rotor between the disc brake pads. This creates friction between the pads and rotor, causing your car to slow down or stop.



Drum Brakes

Drum brakes consist of a brake drum attached to the wheel, a wheel cylinder, brake shoes, and brake return springs. Hydraulic pressure from the master cylinder causes the wheel cylinder to press the brake shoes against the brake drum. This creates friction between the shoes and drum to slow or stop your car.



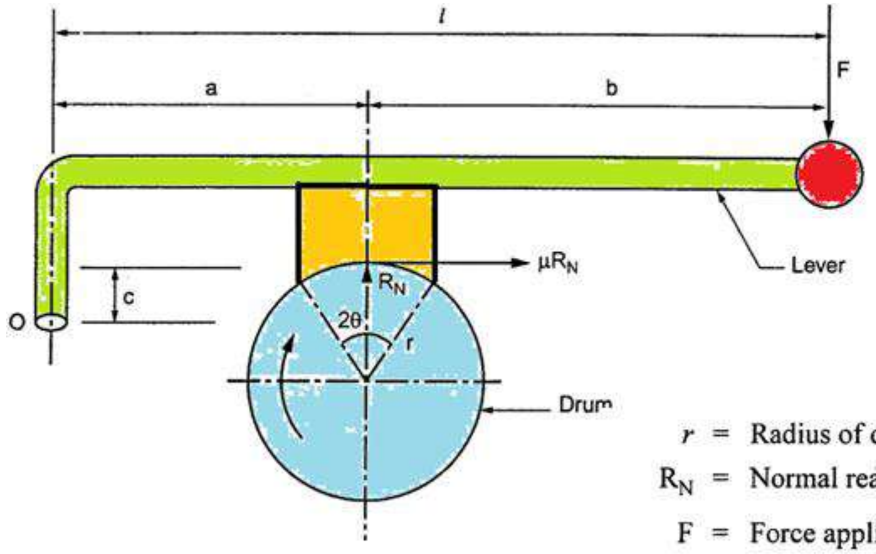
Emergency Brakes

Vehicles also come equipped with a secondary braking system, known as emergency, or parking brakes. Emergency brakes are independent of the service brakes, and are not powered by hydraulics. Parking brakes use cables to mechanically apply the brakes (usually the rear brake).

There are a few different types of emergency brakes, which include: a stick lever located between the driver and passenger seats; a pedal located to the left of the floor pedals; or a push button or handle located somewhere near the steering column. Emergency brakes are most often used as a parking brake to help keep a vehicle stationary while parked. And, yes, they are also used in emergency situations, in case the other brake system fails!

Anti-Lock Brakes

Computer-controlled anti-lock braking systems (ABS) is an important safety feature which is equipped on most newer vehicles. When brakes are applied suddenly, ABS prevents the wheels from locking up and the tires from skidding. The system monitors the speed of each wheel and automatically pulses the brake pressure on and off rapidly on any wheels where skidding is detected. This is beneficial for driving on wet and slippery roads. ABS works with the service brakes to decrease stopping distance and increase control and stability of the vehicle during hard braking.



r = Radius of drum,

R_N = Normal reaction of the block,

F = Force applied at lever end,

μ = Coefficient of friction,

μR_N = Frictional force, and

T_B = Braking torque.

1. Self-locking Brake

. ***When the frictional force is sufficient enough to apply the brake with no external force, then the brake is said to be self-locking brake.***

2. Self-energizing Brake

From equation (ii), it is observed that the moment of applied force ($F \cdot l$) and the moment of the frictional force ($\mu R_N \cdot c$) about O are in the same direction. Thus frictional force (μR_N) helps in applying brake. This type of brake is known as a ***self-energised brake.***

Note : Pivoted Block or Shoe Brake (2θ > 40°)

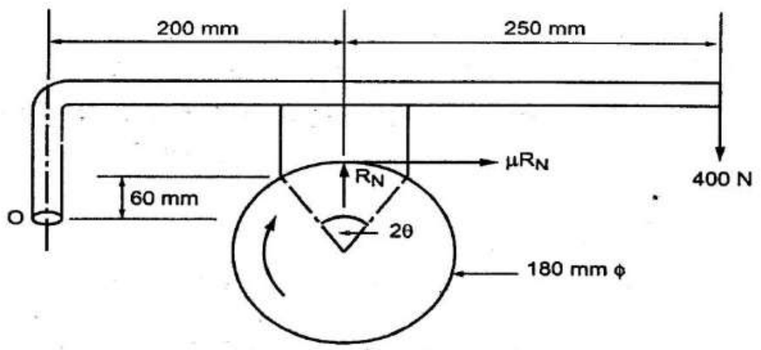
It is assumed that the normal reaction and the frictional force act at the mid-point of the block. Generally angle of contact (2θ) is small. When the value of angle of contact is more than 40° (i.e., when 2θ > 40°), the equivalent coefficient of friction (μ') is used in torque equation.

Equivalent coefficient of friction : $\mu' = \dfrac{4\mu \sin\theta}{2\theta + \sin 2\theta}$

where μ = *Actual coefficient of friction, and*

2θ = *Angle of contact.*

Problem.1. ***A single block brake is shown in Fig. . The diameter of the drum is 250 mm and the angle of contact is 60°. If the operating force of 400N is applied at the end of a lever and the coefficient of friction between the drum and the lining is 0.30, determine the torque that may be transmitted by the block brake.***



Given Data : $d = 180$ mm or $r = 90$ mm $= 90 \times 10^{-3}$ m ;

$2\theta = 60° = 60° \times \dfrac{\pi}{180°} = \dfrac{\pi}{3}$ rad ; $F = 400$ N ; $\mu = 0.30$

☺ ***Solution :*** Since the angle of contact is greater than 40°, therefore equivalent coefficient of friction.

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.30 \times \sin 30°}{\frac{\pi}{3} + \sin 60°} = 0.313$$

Taking moments about the fulcrum O, we get

Down ward = up ward

$$400\,(250 + 200) + \mu' R_N \times 60 = R_N \times 200$$

$$400 \times 450 + 0.313 \times R_N \times 60 = 200\, R_N$$

or $$R_N = 993.3 \text{ N}$$

$$\text{Braking force} = \mu' R_N = 0.313 \times 993.3 = 310.9 \text{ N}$$

Torque transmitted by the block brake, $T_B = \mu' R_N \cdot r = 310.9 \times 0.09$

T_B = **27.98 N-m Ans.** ☜